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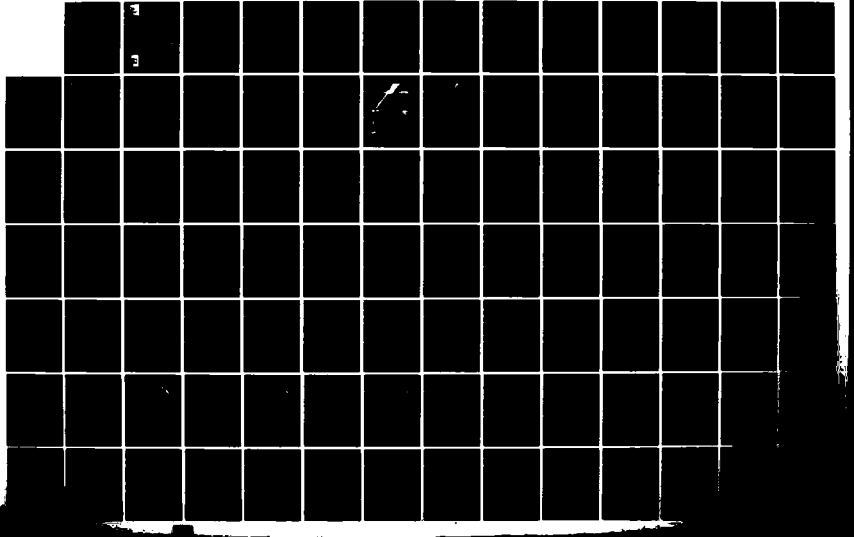

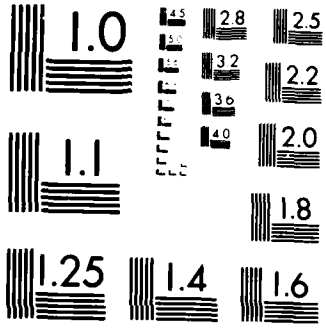

US Army Corps of Engineers

TECHNICAL REPORT HL-80-3

EROSION CONTROL OF SCOUR DURING CONSTRUCTION

Report 5
EXPERIMENTAL MEASUREMENTS OF REFRACTION AND DIFFRACTION DOWNCOAST OF AN OBLIQUE BREAKWATER

by

Lyndell Z. Hales

Hydraulics Laboratory

DEPARTMENT OF THE ARMY
Waterways Experiment Station, Corps of Engineers
PO Box 631
Vicksburg, Mississippi 39180-0631

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When major stone structures are erected in the coastal zone, they alter currents that are in dynamic equilibrium with the existing conditions. Waves breaking on the structures may cause bottom material to be suspended and transported from the region. The result is scour, or erosion, that usually develops along the toe of the structure. To minimize potential cost increases due to scour during construction, it is necessary to quantify the probability and ultimate extent of potential scour during the scheduled

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20. ABSTRACT (Continued).

construction period. Objectives of this research program are to develop techniques to minimize and control scour during nearshore construction, and to predict the probable magnitude of scour that may result as a function of currents and wave climate. One phase of the research program is development of numerical techniques (incorporating both refraction and diffraction effects near the structure) for computing wave-induced velocities, tidal currents, and wave heights in the vicinity of structures, and applying these results to determine sediment transport of the bottom material at the particular site.

The present state of nearshore current and wave theories has reached the point where detailed experimental investigations are required for the verification of analytical developments and numerical models. To provide a foundation for further advancements, a simple beach profile consisting of straight, uniform contours parallel with the shoreline was experimentally studied by Hales (1980). A shore-connected, vertical, thin, impermeable barrier (breakwater) was installed perpendicular to the shoreline to simulate prototype jetties and breakwaters commonly occurring along many coasts. The purpose of the present study is to extend the previous work of Hales (1980) by installing a shore-connected, vertical, thin, impermeable breakwater at a 60-deg angle to the shoreline to simulate a larger range of prototype jetties and breakwaters in existence at the present time. Experimental measurements of refraction and diffraction downcoast of this oblique structure were made to obtain quantitative knowledge of this phenomenon in the lee of the jetty or shore-connected breakwater. These data were then compared with the uniformly valid asymptotic theory of Liu, Lozano, and Pantazaras (1979) for the same arrangement.

The numerical model for determining wave heights downcoast of a straight breakwater at an angle to the shoreline under combined refraction and diffraction, based on the uniformly valid asymptotic theory, was obtained by contract with Dr. Philip Liu, Cornell University. Because the uniformly valid asymptotic theory is developed from the small amplitude wave assumptions, the effect of varying the incident wave height on the wave-height amplification factor, H/H_o, was investigated. The theory and experimental data were found to compare favorably for all wave heights tested in the shadow zone, but diverged with increase in incident wave height farther downcoast out of the shadow zone. While the uniformly valid asymptotic theory is far superior to diffraction theory alone under these conditions, additional numerical work should incorporate nonlinear wave theories for completeness.

PREFACE

The study reported herein was authorized as a part [illegible] Research and Development Program by the Office, Chief of Engineers, US Army. This particular work unit, Erosion Control of Scour During Construction, is part of the Improvement of Operations and Maintenance Techniques (IOMT) Program. Mr. James L. Gottesman was the OCE Technical Monitor [illegible] IOMT Program during preparation and publication of this report.

This study was conducted during the period 1 May 1980 through September 1981 by personnel of the Hydraulics Laboratory of the US Army Engineer Waterways Experiment Station (WES) under the general supervision of Messrs. H. B. Simmons, Chief of the Hydraulics Laboratory; F. A. Herrmann, Jr., Assistant Chief of the Hydraulics Laboratory; R. A. Sager, Chief of the Estuaries Division and IOMT Program Manager; Dr. R. W. Whalin, former Chief of the Wave Dynamics Division; Mr. D. D. Davidson, Chief of the Wave Research Branch; and Dr. J. R. Houston, Research Engineer and Principal Investigator for the Erosion Control of Scour During Construction work unit. The Wave Dynamics Division was transferred to the Coastal Engineering Research Center (CERC) of WES on 1 July 1983 under the direction of Dr. R. W. Whalin, Chief, and Dr. L. E. Link, Jr., Assistant Chief. Dr. L. Z. Hales, Research Hydraulic Engineer, Mr. K. A. Turner, Computer Specialist, Ms. M. L. Hampton, Computer Technician, Mr. R. E. Ankeny, Computer Technician, and Mr. K. M. Strausbaugh, Civil Engineering Technician, performed the experimental portion of the study described herein. The numerical model for determining wave heights downcoast of a straight breakwater at an angle to the shoreline under combined refraction and diffraction, based on the uniformly valid asymptotic theory, was obtained by contract with Dr. Philip Liu, Cornell University. Dr. Hales prepared this report.

Commanders and Directors of WES during the conduct of this investigation and the preparation and publication of this report were COL Nelson P. Conover, CE, and COL Tilford C. Creel, CE. Technical Director was Mr. F. R. Brown.

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CONVERSION FACTORS, US CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

US customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
feet per second	0.3048	metres per second
feet-feet per second	0.0929	metres-metres per second
feet per second per second	0.3048	metres per second per second
pounds-second-second per foot per foot per foot per foot	52.5540137	kilograms-second-second per metre per metre per metre per metre
square feet	0.09290304	square metres

EROSION CONTROL OF SCOUR DURING CONSTRUCTION

EXPERIMENTAL MEASUREMENTS OF REFRACTION AND DIFFRACTION DOWNCOAST OF AN OBLIQUE BREAKWATER

PART I: INTRODUCTION

Statement of the Problem

1. When major structures are erected in the coastal zone, they alter currents that are in dynamic equilibrium with the existing bathymetry. These altered currents may change the existing bathymetry. In addition, waves breaking on the new structure will cause bottom material to be suspended and transported from the region by longshore or other currents. This removal of material from around structures is often not compensated by an influx of additional material; the result is scour, or erosion, that usually develops along the toe of the structure. In order to ensure structural stability, the scour area must be filled with nonerodible material (sufficiently stable to withstand the environmental forces to which it will be subjected). This may result in additional quantities of material being required during construction that can potentially be very costly. To minimize potential cost increase due to scour during construction, it is necessary to quantify the probability and ultimate extent of potential scour during the scheduled construction period.

2. Effective, comprehensive, and low cost procedures do not exist for eliminating scour during construction in the nearshore environment. Determination of potential alternative procedures is seriously hampered by the inability to predict the extent of potential scour. Objectives of the Erosion Control of Scour During Construction research program are to develop techniques to minimize and control scour during nearshore construction, and to predict the probable magnitude of scour that may result as a function of currents and wave climate. One phase of the research program is development of numerical techniques (incorporating both refraction and diffraction effects) for computing the wave field in the vicinity of structures.

3. The present state of nearshore current and wave theories has reached the point where detailed experimental investigations are required for the verification of analytical developments and numerical models. To provide a

firm foundation for further advancements, a simple beach profile consisting of straight, uniform contours parallel to the shoreline was experimentally studied by Hales (1980). A shore-connected, vertical, thin, impermeable barrier (breakwater) was installed perpendicular to the shoreline to simulate prototype jetties and breakwaters commonly occurring along many coasts. This single jetty (shore-connected breakwater) simplified the experiment, facilitated direct comparisons with numerical model results, and provided greater understanding and insight into the phenomenon of wave-height variations downwave of a breakwater than would a more complex geometry.

4. Analytical models of wave fields surrounding shore-normal breakwaters have been developed based on asymptotic theory and the parabolic approximation (for example, Liu and Mei 1975, 1976; Liu, Lozano, and Pantazaras 1979; and Lozano and Liu 1980). These analytical models have been compared with the experimental data of Hales (1980) by Liu (1982) and Tsay and Liu (1982), and the overall agreement between theory and experiment was considered to be good. Knowledge of these important phenomena can be used as the basis for advanced studies of sediment movement around major structures under combined effects of refraction and diffraction, when the structure is oriented perpendicular to the shoreline.

Objective and Scope of the Study

5. The objective of the present study was to extend the previous experimental work of Hales (1980) by installing a shore-connected, vertical, thin, impermeable barrier (breakwater) at a 60-deg angle to the shoreline to simulate a larger range of prototype jetties and breakwaters in existence and to test a new theory of waves in the lee of a structure. The experimental measurements were made to obtain quantitative knowledge of combined refraction and diffraction in the lee of a jetty or shore-connected breakwater. These data were then compared with the uniformly valid asymptotic theory of Liu, Lozano, and Pantazaras (1979) for the same arrangement, thus providing verification data for this numerical approximation which has not been previously available. This report presents details of the experimental investigation to measure combined refraction and diffraction in the lee of a jetty or breakwater. A literature review of the theory of refraction and diffraction is presented. In addition, a new uniformly valid asymptotic theory of combined

PART II: EXPERIMENTAL INVESTIGATION

Experimental Layout

6. This study to investigate wave heights downcoast of an obliquely oriented shore-connected breakwater or jetty under the combined effects of refraction and diffraction was conducted in the experimental facilities of the US Army Engineer Waterways Experiment Station (WES). The experimental arrangement, which was molded in cement mortar, consisted of a 50- by 60-ft* area, with a water depth of 1 ft in the open-ocean region (Figure 1). The

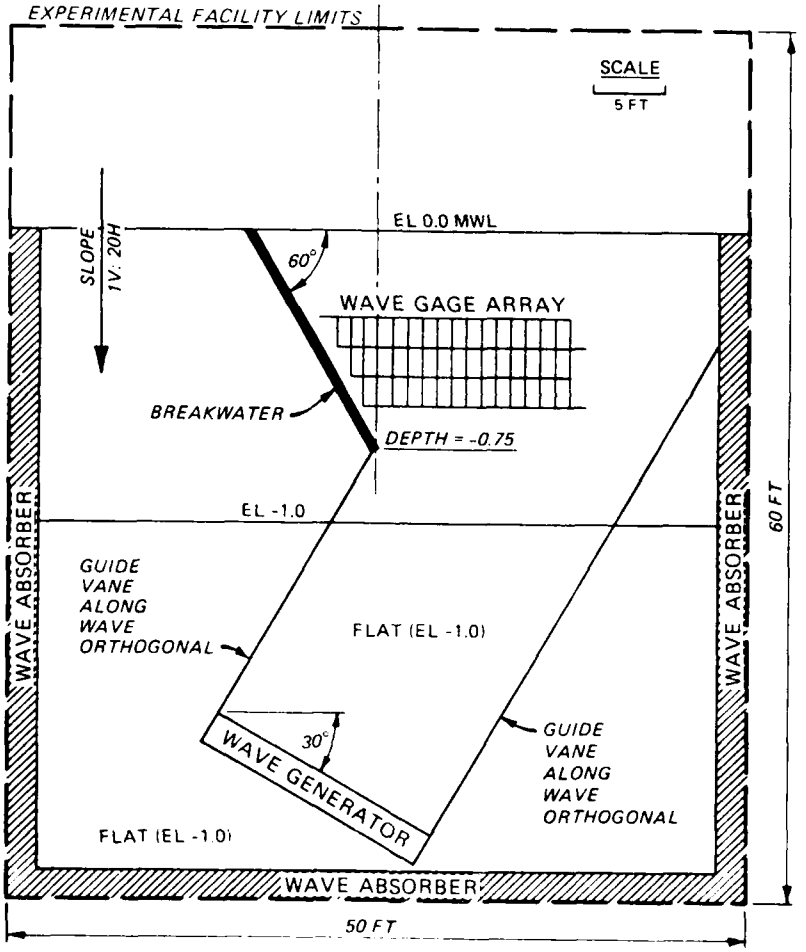


Figure 1. Experimental layout

* A table of factors for converting US customary units of measurements to metric (SI) units is presented on page 3.

9. The ADACS are capable of automatically calibrating the wave sensors, controlling wave generators, acquiring data from the sensors at a high sampling rate, and analyzing test data. Data are taken and recorded on disc or magnetic tape for direct analyses by the minicomputer system or on magnetic tape in a format compatible with a Honeywell DPS1 for backup analyses. Automatic calibration of wave sensors has reduced the time required to calibrate the sensors by a factor of four. In addition, several times the number of tests can be run during a day with test results analyzed at completion of model tests by minicomputer. The system configuration (Figure 2) of ADACS consists of the following subsystems: (a) digital data recording and controls; (b) analog recorders and channel selection circuits; (c) wave sensors and interfacing equipment; and (d) wave generators and control equipment.

Wave sensors

10. The data acquired from wave experiments are the water-surface

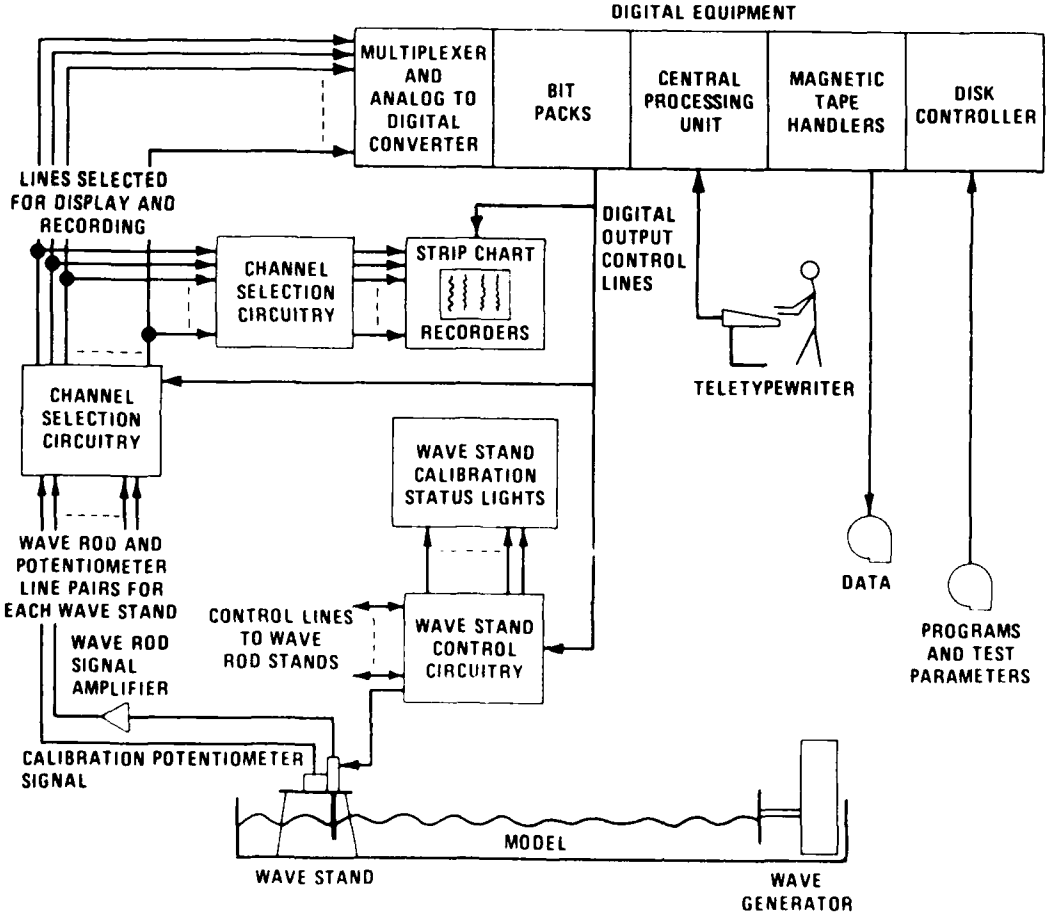
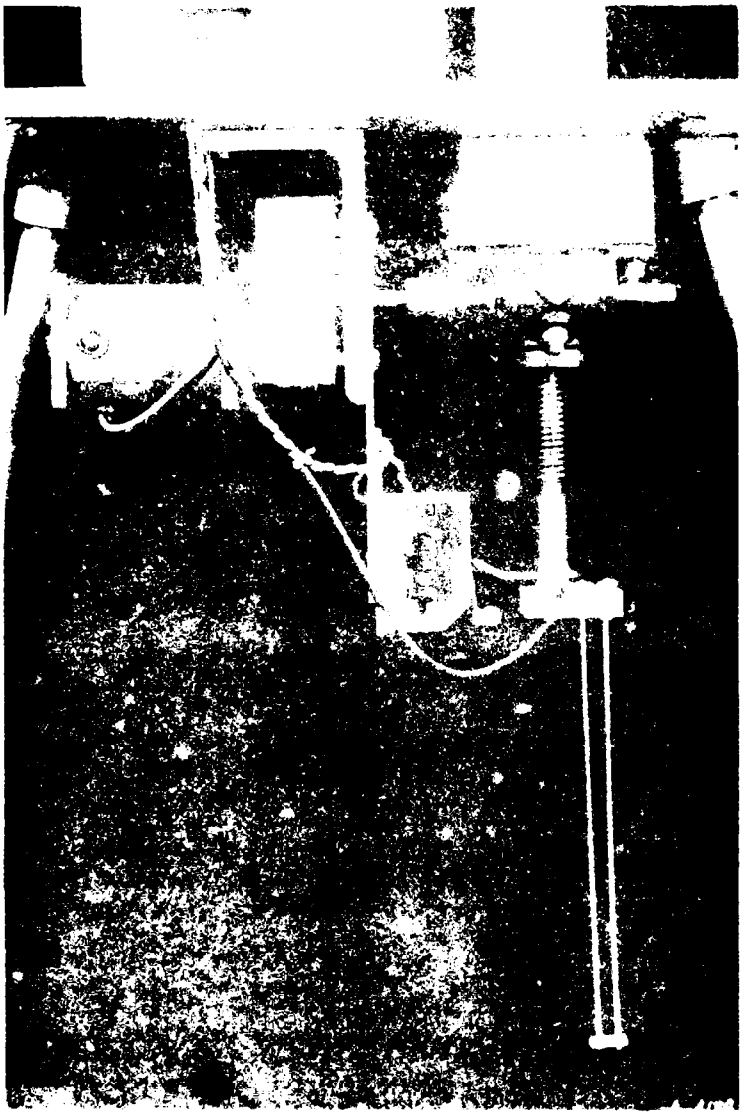


Figure 2. Schematic of components of ADACS



variations about a reference water level. This information is collected at selected geographic locations within the experimental facility for specified wave conditions at the generator. Wave sensors are used to obtain this information at selected locations in the facility. Each of the water-surface-piercing, parallel rod wave sensors is connected to a Wheatstone bridge, and a transducer measures the conductance of the water between the two parallel rods that are mounted vertically (Figure 3). The conductance is directly proportional to the depth of submergence of the two rods in the water. The output of each wave sensor is routed through shielded cables to its signal conditioning equipment where it is processed for recording. The

[illegible] elevations to an accuracy of 0.001 ft.

[illegible] supplies and good signal-to-noise

[illegible] the wave sensor bridge maintains

[illegible] e elevation data in millivolts

[illegible] sensor must be calibrated. The

[illegible] sensors (maximum of 25 rods

[illegible] vided by ADACS. To calibrate

each set of parallel rods, the voltage from the signal conditioning equipment is monitored and recorded as the parallel rods are moved vertically a known distance into or out of the water. A precision, linear-position potentiometer is located on the wave sensor stand and is coupled directly to the parallel rods by a gear train driven by an electric motor. By moving vertically the coupled wave sensor and potentiometer with the electric motor and by monitoring the output voltage from the potentiometer, the wave sensor can be moved vertically a precise distance. The electric motor for each wave sensor is controlled by a control/sense line and a relay contact. The minicomputer controls the vertical movement of each wave sensor by actuating the control/sense line. The central processing unit acts as a voltage comparator by monitoring the potentiometer voltage and comparing it with a reference voltage which is determined from desired displacement and potentiometer calibration. When the voltage comparison is satisfied, the control/sense line is reactivated, the electric motor stops, and voltage samples from the rods and potentiometers are acquired. By systematically moving each wave sensor through 11 quasi-equally spaced locations over the range of rod length used, voltage versus known displacements are obtained from which a calibration curve for each sensor can be calculated and recorded on magnetic tape or disc. After collecting the calibration data, the minicomputer analyzes these data by least-squares fitting a set of curves (linear, quadratic, or spline) to the data, determining the best order of fit, and comparing the maximum deviation of the best fit with a previously acceptable value for this maximum deviation.

Data acquisition and analysis

12. During the acquisition mode, wave data for a specified wave condition at the wave generator are collected from a maximum of 50 wave sensors, recorded on analog strip charts, digitized, and recorded on magnetic tape or disc for further analyses. The sampling scheme is flexible and can be tailored for different applications with maximum throughput rates theoretically limited by the multiplexer rate and allocatable buffer size. The sampling scheme used in this investigation was 60 discrete voltage samples equally spaced over each wave period for a predetermined number of 90 wave periods for each of the sensor locations. The minicomputer calculated from input parameters the lag at the beginning of data acquisition by 10 wave periods after starting the generator, provided timing pulses for synchronizing and controlling the recorders, and determined completion of the test. The determination of the height of

each wave of the monochromatic wave train was performed (at each sensor location), the average of these 90 individual heights was calculated, and the standard deviation of these individual observations about the mean was computed. The value ultimately determined as the wave height at each sensor location was this mean value plus or minus one standard deviation.

Wave Gage Locations

13. The wave-height data downcoast of the experimental breakwater that was positioned at a 60-deg angle with the shoreline were obtained by operating 36 wave-height sensors simultaneously for each test condition. Two of these wave-height sensors were located in the deeper water (1 ft deep) near the wave generator to ascertain the initial generated wave height. The remaining 34 sensors were positioned along four lines parallel with the shoreline at distances of 6, 8, 10, and 12 ft from the shoreline (Figure 4); the still-water depths at these four sections were 0.3, 0.4, 0.5, and 0.6 ft, respectively. The wave gages were placed on a supporting platform such that only the wave sensor probe penetrated the water surface, thereby eliminating any local disturbance caused by instrument stands touching the water surface. A dry bed photograph of the experimental facility with the experimental breakwater in place is shown in Figure 5.

14. In order to compare experimental results with numerical or analytical investigations, it is necessary to have good definition of the experimental data. The 34 wave-height sensors were initially placed at 2-ft intervals along the four sections parallel with the shoreline, and the complete set of experimental data was obtained (gage arrangement No. 1, Figure 6). To provide a more dense data display, the entire physical arrangement of the wave sensors was displaced laterally along the four section lines for a distance of 1 ft, and the same set of wave data (period and height) was repeated. This second testing arrangement is shown as gage arrangement No. 2, Figure 7. All data from these two gage arrangements resulted in a data set that presented the wave heights at 1-ft increments along the four sections which run parallel with the shoreline, with the data extending from very near the breakwater deep in the shadow zone and extending across the region where the waves experience both refraction and diffraction. These precise experiments provide data that define the wave height downcoast of the structure and can be used for comparison with numerical or analytical studies of the same concept.

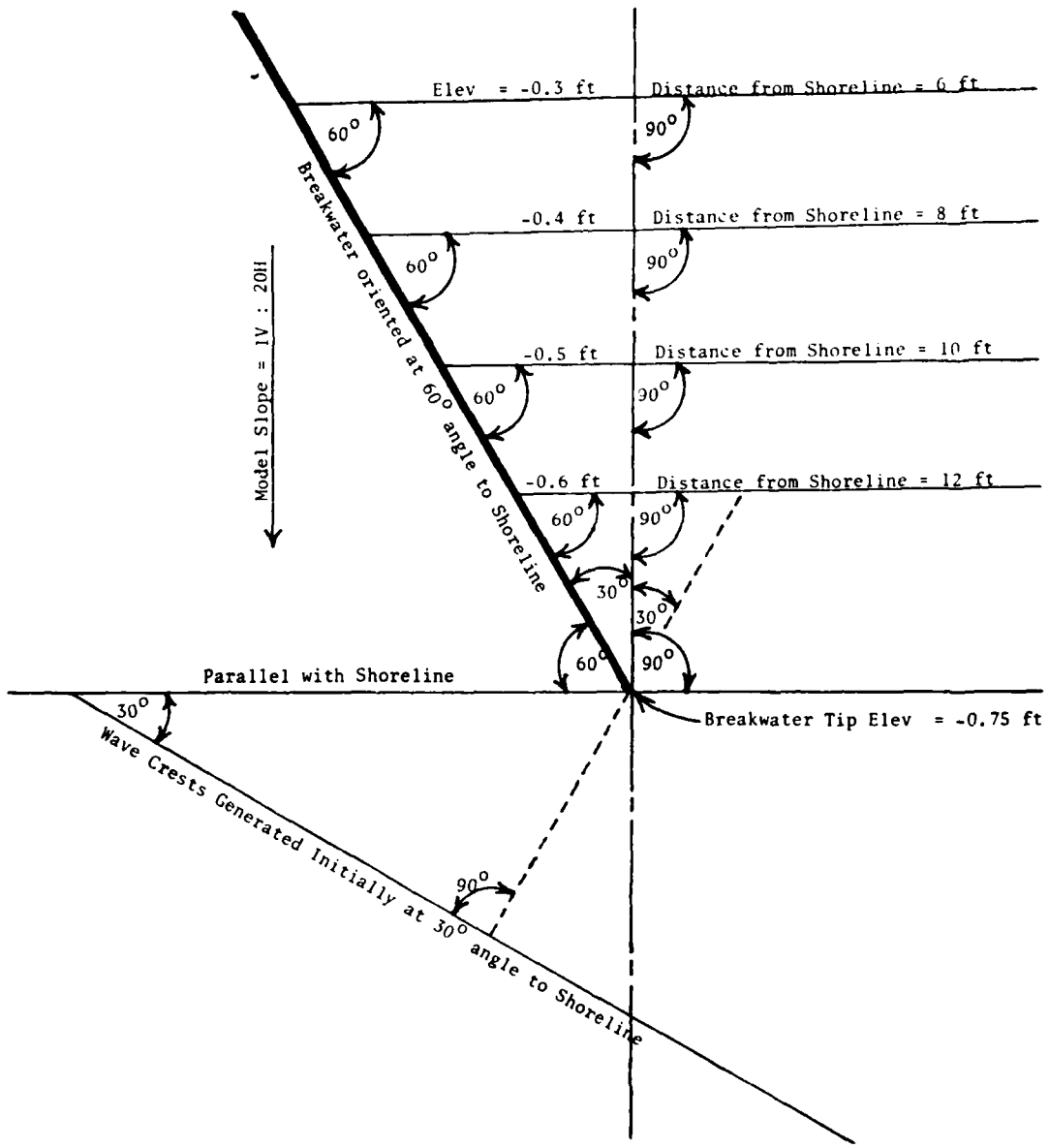
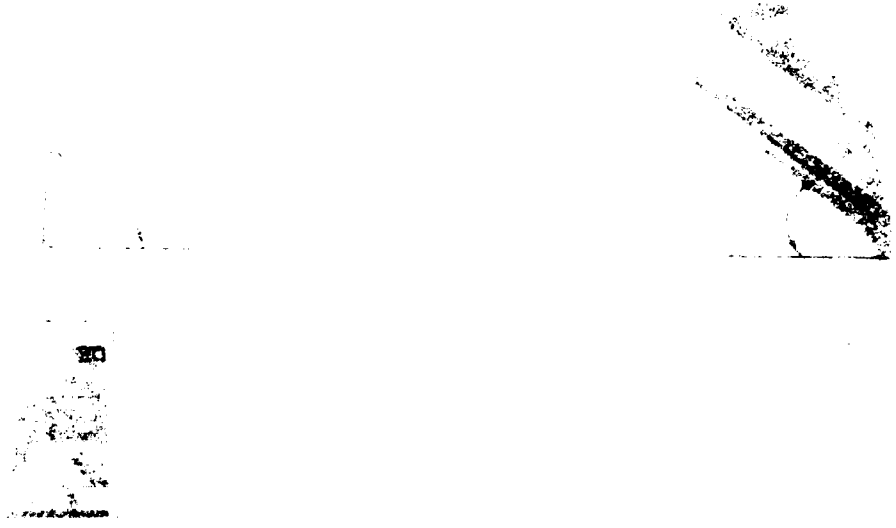


Figure 4. Experimental layout showing location of four sections parallel with shoreline along which wave-height sensors were positioned



Aerial photograph of experimental facility with breakwater in place, showing four sections where wave-height sensors were positioned and elevation of each section

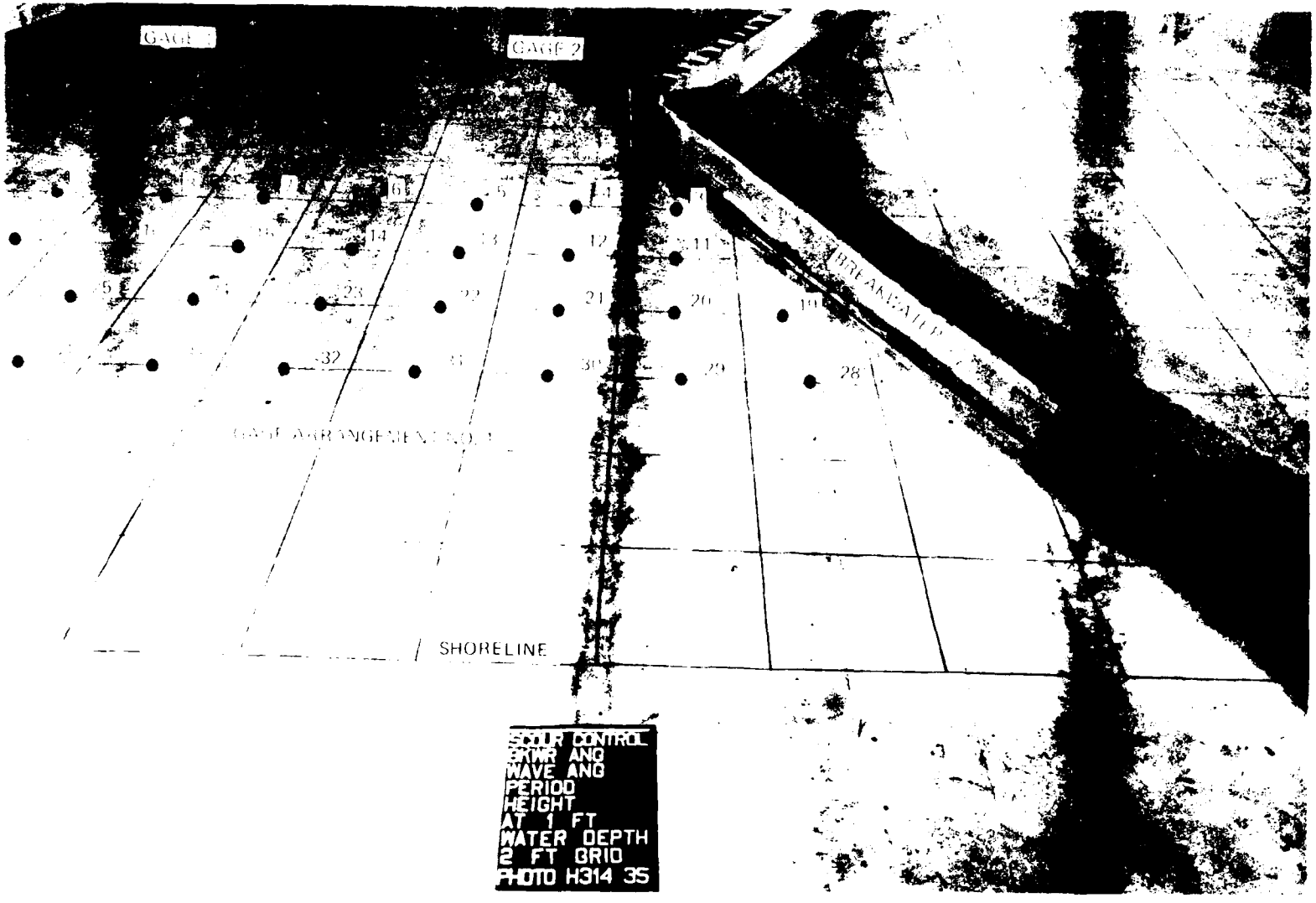


Figure 6. Wave gage arrangement No. 1, showing location of wave-height sensors

Figure 2. Wave gage arrangement No. 2, showing location of wave-height sensors

PART III: LITERATURE REVIEW OF COMBINED REFRACTION AND DIFFRACTION

15. The approximate solution of water wave refraction caused by a variable bathymetry is well known and can be derived by assuming that bottom reflections are negligible or by the more rigorous Wentzel-Kramers-Brillouin (WKB) approximation. The exact solution for the diffraction of surface waves by vertical barriers of simple cross section in water of constant depth also is well known (being analogous to classical problems of physics). However, an analytical theory for the practical case of combined refraction and diffraction has not been completely developed. The present engineering practice for determining wave heights under this condition is a stepwise procedure (Dunham 1951, Liu and Mei 1976, US Army Coastal Engineering Research Center 1977). The procedure involves the following steps: (a) calculate refraction effects up to the barrier (jetty or shore-connected breakwater); (b) calculate for a "few" wavelengths the effects of diffraction, assuming a constant water depth; and (c) beyond this region calculate the refraction effects only. This procedure is obviously imprecise, and Mobarek (1962) indicates that the method is suitable only for intermediate water depths. In addition, Whalin (1972) states that this procedure is only valid for small refraction effects.

Water Wave Refraction

16. In intermediate and shallow water, the phase speed of a surface gravity wave depends on water depth. Since wave celerity decreases as depth decreases, phase velocity varies along the crest of a wave propagating at an angle to underwater contours because that part of the wave in deeper water moves faster than that part in shallower water. This variation causes the wave crest to bend toward alignment with the contours. This bending effect due to changes in bottom topography, called refraction, depends on the relation of water depth to wavelength, d/L, and is analogous to refraction of other types of waves such as light. A basic assumption in wave refraction theory is the conservation of energy between wave orthogonals (i.e., no diffraction of energy along wave crests). The change in wave direction of different parts of the wave results in convergence or divergence of wave energy and materially affects the forces exerted by waves on structures and of the

capacity of waves to transport sand either alongshore or onshore/offshore.

17. Procedures for the computation of refraction of surface gravity waves on water of nonuniform depth involve the assumption that a wave with a curved crest pattern and variable amplitude along the crest behaves locally as a straight-crested wave of constant amplitude. Rayleigh (1877) appears to have been the first to use the approximations of geometrical optics in this analysis, and theoretical results have been developed with respect to energy flux and phase speed. As expressed by Keller (1958), the geometrical optics theory defines a propagation velocity at each point on the water surface, with this velocity being exactly that which waves of given period would have in water of uniform depth at *all* points. By employing Fermat's principle of optics, wave rays are defined and surface waves are assumed to propagate along these rays. The variation of the amplitude along the rays is determined by the use of the principle of conservation of energy. This principle (in its optical form) states that the flux of energy is the same at all cross sections between two adjacent wave rays. The energy flux is proportional to the square of the amplitude of the waves and to the distance between the rays, and hence the wave-height variation along the ray is available.

18. Keller's (1958) derivation is based upon an "asymptotic" solution of the equations of the exact linear theory for periodic waves *in water of* arbitrary nonuniform depth. The solution is asymptotic in the sense that the depth and wavelength are small compared with the horizontal scale of the bottom contours. The first term of the solution agrees exactly with the asymptotic form of the solution for waves in water with a uniformly sloping bottom as the bottom slope tends to zero. This solution conforms with all the principles of the geometrical optics theory of Rayleigh (1877) and thus provides a derivation of that theory. The results are not valid, however, at caustics or ray crossings.

19. In problems of linear wave propagation over mild slopes, the principle of geometrical optics has been applied by Carrier (1966) as the first approximation in a systematic perturbation scheme while the bottom is considered to be locally horizontal. The depth variation was dealt with afterward by requiring the appropriate energy conservation. This was also the basic idea for the work of Koh and LeMéhauté (1966) in which the transformation of progressive waves was investigated as they travel from deep water to shore. The Stokes' theory at a fifth order of approximation was applied along

with the method of conservation of energy flux. It is assumed that the wave is simply harmonic in time. The first, third, and fifth orders of approximation were compared with each other and with experiments. The differences between the predictions of wave-height changes based on the three orders of approximation were found to be small, on the order of 5 percent. For practical purposes, the third-order theory was found to give reliable results. The third and fifth order Stokes' theories are based on a series expansion in terms of H/L where terms of the order of $(H/L)^3$ and $(H/L)^5$, respectively, are retained and higher order terms are neglected. It should be noted that this theory is based on an expansion in term of the wave steepness, H/L, and consequently can be expected to better approximate limiting steepness waves in deep water. However, it cannot be expected to adequately predict wave characteristics in shallow water, since water depth is not a parameter in the series expansion. Thus the theory is a finite amplitude deepwater wave theory.

20. In cases of limiting shallow water, the wave conditions are nearly independent of wavelength, and the important parameters are water depth and the ratio of wave height to water depth. Keulegan (1950) showed that Stokes waves are most nearly valid in water deeper than about $d/L > 1/8$ to $1/10$. In shallower water, cnoidal wave theory appears to be more satisfactory, and Masch (1964) investigated the problem of wave shoaling using cnoidal wave theory with the formulas developed by Keulegan and Patterson (1940). Masch (1964) assumed hydrostatic pressure distribution and neglected the convective inertia term in his expression for the energy flux. The third and fifth order cnoidal theories are based on a series expansion in terms of H/d, where terms of the order of $(H/d)^3$ and $(H/d)^5$, respectively, are retained and higher order terms neglected. It should be noted that this theory is based on an expansion of the relative wave height (H/d) and can be expected to better approximate the wave form in shallow water. However, it cannot be expected to do a very good job of approximating the wave form for limiting steepness waves in deep water. In that case, water depth is unimportant and wavelength is crucial. This theory could be considered a finite amplitude intermediate and shallow-water theory.

21. A technique of "asymptotic expansion" was applied by Mei, Tlapa, and Eagleson (1968) to water wave propagation over an uneven bottom that has straight and parallel contours. Attention was focused on the establishment of a rigorous scheme of successive approximation for higher order corrections.

the bottom depth was assumed to vary [illegible] ing a compressed coordinate in the direction normal to the contour and by assuming an expansion of the WKB type, the weakness of the depth variation in the normal direction was incorporated in the mathematical formulation. The conventional linearized theory of wave refraction was obtained as the first order solution without the explicit assumption of Snell's law. In the second order, a steady-state depression of the mean water surface was found for the general case where the incident wave approaches the coast obliquely. In the first order, this development was the same as the classical theory of Kajiura (1877). At higher order, some differences existed from the usual adaptation of the Stokes' theory for a horizontal bottom. The cause of the differences is the explicit appearance of the variation of the local boundary conditions for the first harmonic at second and higher orders. The theory is expected to hold up to the neighborhood where the wave breaks, except at the shoreline where a singularity exists for all orders.

22. The refraction of surface gravity waves propagating in an ideal fluid was investigated by Battjes (1968) with amplitudes small so that linear theory was applicable and harmonic in time. It was known a priori that the velocity of propagation of a wave crest to the 1/2 order and greater is a function of the wave height. In zones of convergence or divergence of wave energy, gradients in wave height exist at the portions of the wave crests. The portions of greater wave height will propagate faster than the portion of lesser wave height and this will, in turn, create bending of the wave crest or a refraction to that bending caused by the bottom topography, called refraction. This supplemental bending is not usually accounted for in refraction analysis. Battjes (1968) developed an expression for this bending relative to the wave crest because of wave-height variations along the crest from an exact derivation of the wave number, k. It was determined that the assumption of the exact wave number, k_{exact}, to plus correction [illegible]

[illegible]

[illegible] is the phase speed [illegible]

$$\left(\frac{c}{c_g}\right)\sin\alpha\,\frac{\partial}{\partial s}\left(\frac{1}{k}\frac{\partial k}{\partial s}\right)$$

where

g = gravitational constant, 32.174 ft/sec^2

k = wave number, $2\pi/L$, 1/ft

d = local water depth, ft

a = local wave amplitude, ft

The second derivative of the wave amplitude in the horizontal plane is given by a_{xx} or a_{yy} . The rate of power transmission P , or energy flux, was determined to be:

$$P\Delta b = \left(\frac{1}{2}\ \rho g a^2 n\ \frac{\omega}{k}\ \Delta b\right)\left(1 + \frac{a_{xx} + a_{yy}}{k^2 a}\right)^{1/2} \tag{2}$$

where

Δb = wave ray spacing, ft

ρ = fluid density, lb-sec^2/ft^4

n = ratio of group velocity c_g , to phase velocity c , dimensionless

ω = angular frequency, $2\pi/T$, 1/sec

Define:

$$\delta = \left(\frac{a_{xx} + a_{yy}}{k^2 a}\right) \tag{3}$$

The commonly used existing procedures (Dunham 1951, Liu and Mei 1976, CERC 1977) for construction or computation of refraction diagrams utilize phase speeds that are obtained by neglecting δ .

23. Battjes (1968) examined the omission of δ from a qualitative standpoint. In an area of strong local convergence, omission of δ from Equation 1 results in underestimating the local phase speed. The result is that in Equation 2 the ray separation Δb will be underestimated. Thus there are generally two contributions to the error that results in the computed wave amplitude a . However, these two contributions are of a different nature because the effect on the wave pattern of using an incorrect value for the phase speed is cumulative, whereas the effect on energy flux is local. In any case, the omission of δ will generate wave heights at variance with the height inferred from refraction diagrams based on linear small amplitude wave theory which neglects the effects of wave-height gradients along the wave crest, so that it would appear that energy had been transferred across orthogonals. An estimate of the magnitude of δ has been approximated by Battjes (1968) for four

distinctly different cases. For simple shoaling, the amplitude varies only in the direction of wave propagation, and for shallow water the variation was:

$$\delta = 8 \times 10^{-3}\left(\frac{L}{d}\right)^2 s^2 \qquad (4)$$

where

L = local wavelengths

s = bottom slope

For short-crested waves where the distance along the wave crest is two or three times the wavelength, δ = -20 percent or -10 percent, respectively. For the case of diffraction around a semi-infinite breakwater, δ was found to reach values between +10 percent and -10 percent at points a distance of one wavelength from the breakwater tip, decreasing inversely proportional to the distance from the tip. For the case of diffraction through a gap of width two times the wavelength, δ was evaluated in a few points on the center line of the gap where it was found to reach values of 25, 7, and 3 percent at distances of 1, 2, and 4 wavelengths, respectively, from the gap.

Water Wave Diffraction

24. Diffraction of water waves is the phenomenon by which wave energy propagates into the sheltered lee of structures even in the absence of bathymetric refraction. In these situations, wave crests bend (even in constant depth water) and gradients of wave height exist along the wave crest. This phenomenon is most visible when a train of regular waves is interrupted by an obstruction such as a jetty or shore-connected breakwater. The theory of water wave diffraction can be explained by Huygens' principle. Each point of an advancing wave front (wave crest) may be considered as the center of a secondary circular wave which advances in all directions. The resultant shape of the crest is the envelope of all these secondary waves. In a straight-crested wave train, the envelope of the secondary waves is a straight line also. When the wave passes an obstruction, the energy intensity at a certain point is a vector combination of all the circular waves emitted by every point of the passing wave train.

25. Sommerfeld (1896) presented a solution for the diffraction of light waves past the edge of a semi-infinite screen. Penny and Price (1944) showed that this is also the solution of the water wave diffraction problem at the

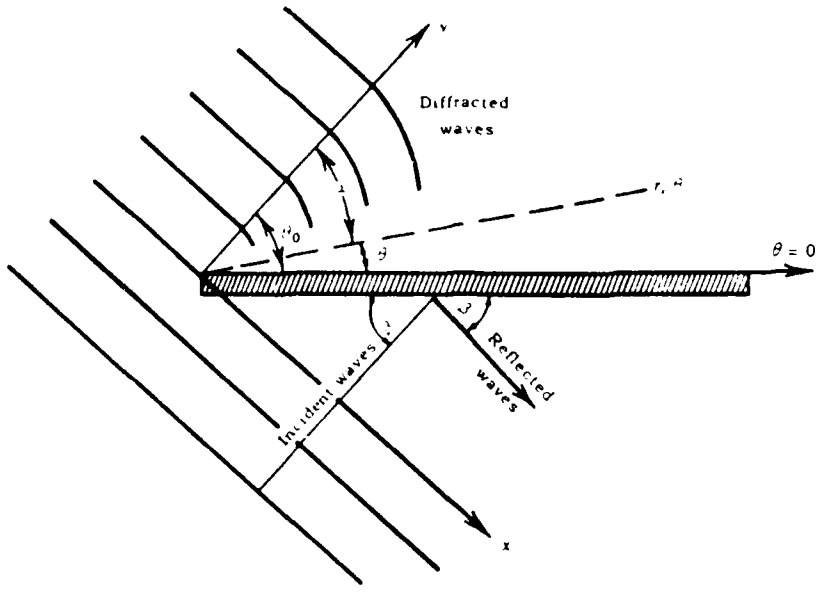


Figure 8. Definitive sketch, wave diffraction around a breakwater

end of a semi-infinite obstacle such as a jetty or shore-connected breakwater. This exact solution of the surface elevations behind the breakwater is applicable only to water of constant depth and waves of small amplitude. Putnam and Arthur (1948) summarized the solution of Penny and Price (1944) for the definitive sketch of Figure 8. In cylindrical coordinates, the water-surface elevation is:

$$\eta = \left(\frac{aikc}{g}\right) e^{ikct} \cosh kd \, F(r,\theta) \tag{5}$$

where

η = water-surface elevation, ft

i = square root of -1

and the other symbols have been previously defined. $F(r,\theta)$ is a function which satisfies the wave equation in cylindrical coordinates:

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2} + k^2 F = 0 \tag{6}$$

In the presence of a jetty or breakwater, the boundary condition is imposed so that the normal component of the fluid velocity is zero along the breakwater, leading to the solution:

$$F(r,\theta) = \frac{1}{\sqrt{2}} e^{i[(\pi/4)-kr \cos(\theta_o-\theta)]} \int_{-\infty}^{u_1} e^{-(i\pi u^2/2)} du$$

$$+ \frac{1}{\sqrt{2}} e^{i[(\pi/4)-kr \cos(\theta_o-\theta)]} \int_{-\infty}^{u_2} e^{-(i\pi u^2/2)} du \quad (7)$$

$$u_1 = 4\sqrt{\frac{kr}{\pi}} \sin\left[\frac{1}{2}(\theta_o - \theta)\right] \quad (8)$$

$$u_2 = 4\sqrt{\frac{kr}{\pi}} \sin\left[\frac{1}{2}(\theta_o + \theta)\right] \quad (8)$$

Bretschneider (1966) has presented computational procedures for evaluating the diffraction coefficients at arbitrary points behind jetties or breakwaters.

26. Wiegel (1962) developed a graphical procedure for determining diffraction coefficients of waves passing the tip of single breakwaters. The family of diagrams shows, for uniform water depth, lines of equal wave-height reduction displayed in terms of the diffraction coefficients. The diffraction diagrams (typical example, Figure 9) are constructed in polar coordinate form centered at the structure tip. The arcs behind the breakwater are spaced one radius-wavelength unit apart so in application, a specific diagram must be scaled up or down so that the particular wavelength corresponds to the scale of the hydrographic area under investigation. The set of diffraction diagrams of waves passing the tip of a single breakwater was presented by CERC (1977). Figure 9 is the configuration of the wave approach direction analogous to the physical hydraulic model layout used in this experimental investigation.

27. Laboratory tests were performed by Harms (1979) to investigate the distribution of wave heights in the lee of a breakwater (shore-connected) for waves normally incident upon the structure and with a horizontal bottom both in front of and in the lee of the structure. In general, satisfactory agreement was obtained between measurement and theory, but diffraction theory was not found to be conservative. At large distances in the shadow zone, measured wave heights consistently exceeded theoretical values. Close to the breakwater outside the shadow zone, the measured maximum wave height was also found to be larger than that predicted by theory. The diffraction behavior appeared

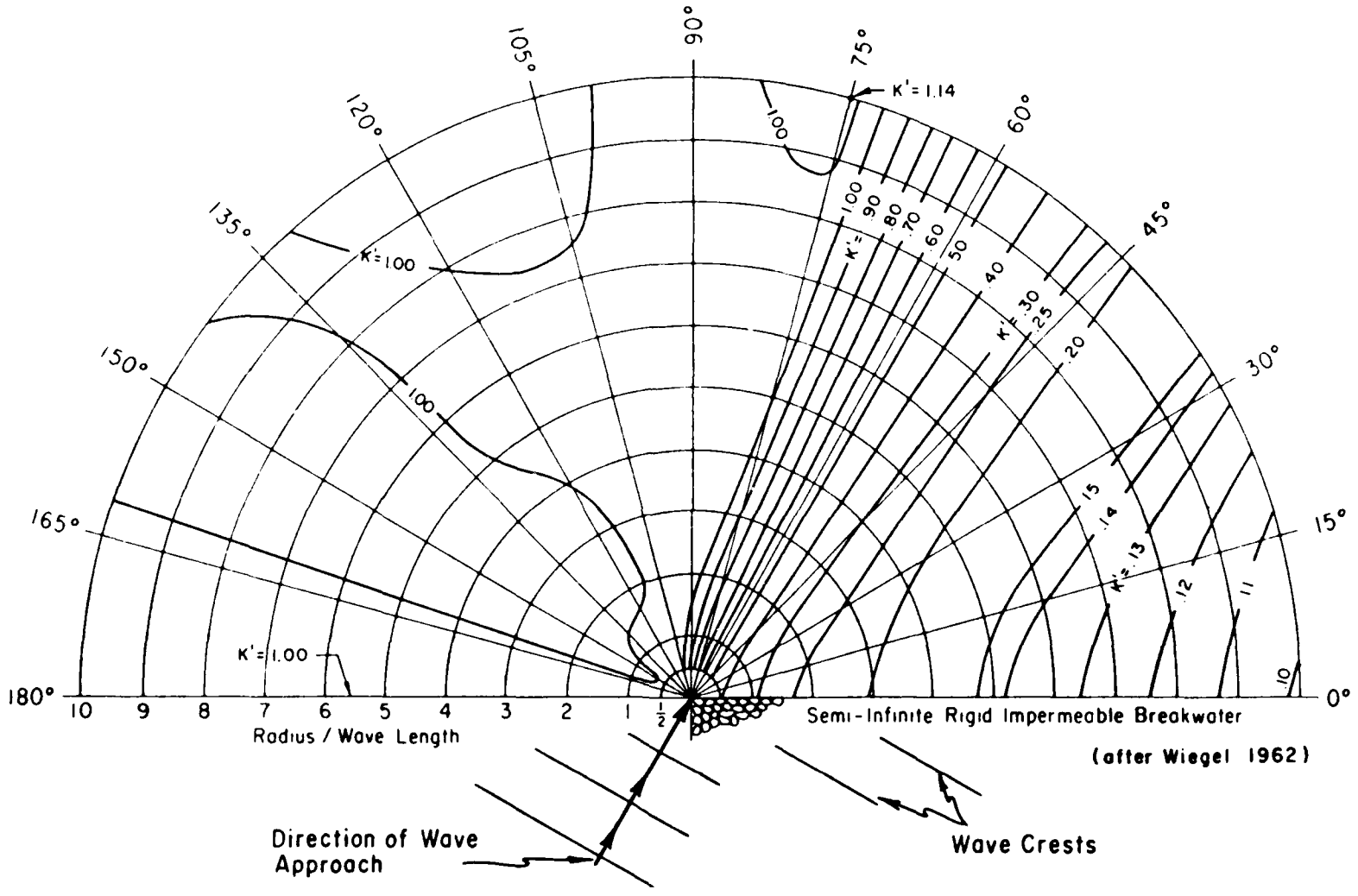


Figure 9. Typical example, wave diffraction past a semi-infinite breakwater, angle of incidence = 60 deg. This configuration is analogous to the physical hydraulic model layout of this experimental investigation

to be insensitive to the intensity of wave reflections from the seaward side of the breakwater.

Combined Water Wave Refraction and Diffraction

28. The bathymetry shoreward of a jetty or breakwater usually is not flat or even uniform; hence, refraction generally occurs in addition to the diffraction effects. While a general unified analytical approach to the simultaneous solution of these two distinctly different phenomena has not been entirely developed, considerable insight has been gained through the theoretical work of Liu and Mei (1975, 1976), Lick (1978), Liu and Lozano (1979), and through the earlier experimental work of Mobarek (1962). The procedure usually followed by coastal engineers concerned with wave-height variation behind jetties or breakwaters is to construct refraction diagrams shoreward to the structure, then construct diffraction diagrams for three or four wavelengths shoreward of the jetty, and finally refract the last wave crest on toward the shoreline. This procedure is schematized in Figure 10 where the overall refraction-diffraction coefficient, K_{r-d}, in the region behind the structure is:

$$K_{r-d} = K_r K_d \sqrt{b_1 b_2}$$

where

K_r = refraction coefficient at the structure, dimensionless

K_d = diffraction coefficient on last wave crest behind the structure from which additional refraction computations are performed, dimensionless

b_1 = orthogonal spacing at the last diffracted wave crest, ft

b_2 = orthogonal spacing near the shore, ft

29. Mobarek (1962) experimentally investigated the effect of bottom slope on wave diffraction through a gap in a breakwater normal to the incident wave direction. Also investigated was the effect of an abrupt increase or decrease in the water depth behind the breakwater. The theoretical analysis for the comparison of experimental results followed the treatment of Penny and Price (1944) restricted to the case of normal incidence for which the Sommerfeld (1896) solution is simplified and in the presence of a horizontal bottom. Two fundamentally different basin configurations were used in the study. The

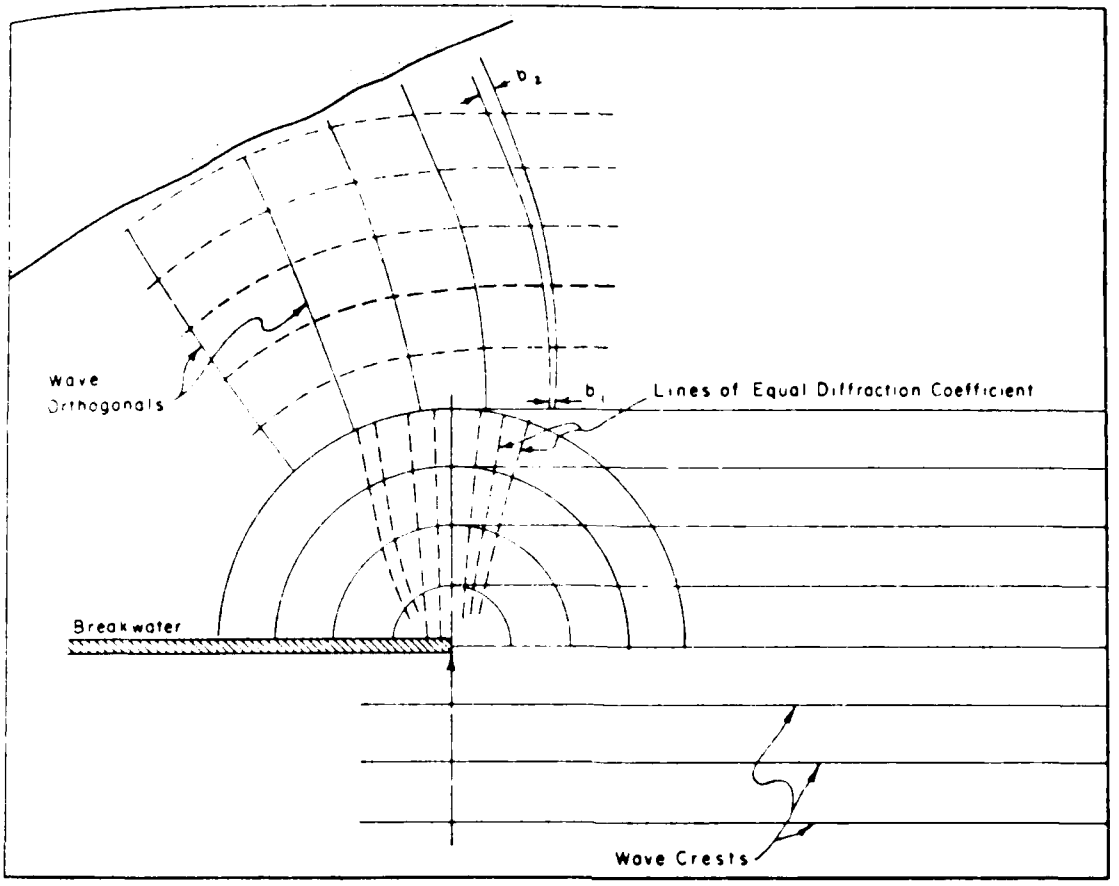


Figure 10. Graphical procedure for determining overall refraction and diffraction effects past a breakwater (after CERC 1977)

first consisted of a longitudinally sloping bottom with the slope commencing at the breakwater and extending to the shoreline. The second was constructed with a flat bottom extending beyond the breakwater gap but sloping laterally to the shoreline. Taking into consideration the serious limitations of the experimental equipment (very small model, 72 sq ft), the investigation led to the conclusion that the procedure usually followed for estimating wave heights behind jetties or shore-connected breakwaters (Figure 10) was sufficiently good for medium period waves; but in the case of long waves, the effect of the shoaling bottom on waves should be taken into consideration. Experiments on a larger scale were highly recommended.

30. For a long shore-connected breakwater on a slowly varying bottom, an asymptotic theory has been developed by Liu and Mei (1976) that accounts for the combined effects of refraction and Fresnel diffraction of water waves. However, for short jetties or groins, the reflection and diffraction effects

[illegible] approximation is a combination of Kirchhoff [illegible] permits the diffraction field near the [illegible] and this can, in turn, be used [illegible]

[illegible] that accounts for the combined effect of refraction due to [illegible] and diffraction by a long shore-connected breakwater [illegible] because this theory is valid near the edge and the tip of the breakwater. The wave behavior in the near field is of particular interest for studying the scour and erosion that may occur near the tip of a breakwater. In this analysis, recent developments in the field of formal [illegible] have been included to extend the effect of diffraction.

[illegible] and Sprinks (1976) [illegible] equation that governs short-wave propagation [illegible] Berkhoff's expression, [illegible] the mild-slope wave equation:

$$[illegible] \qquad (10)$$

where

[illegible]

$$[illegible] \qquad (11)$$

where

ϕ = [illegible]

$\vec{u}$ = [illegible]

Equation 11 reduces to the diffraction Helmholtz equation in deep or constant-depth water. In shallow water, the equation reduces to the linear long-wave equation.

33. Houston (1980) solved Equation 11 by the use of a hybrid finite element numerical model originally developed by Chen and Mei (1974) to solve the diffraction Helmholtz equation in a constant-depth region. The appropriate modifications, including variable depth and frequency dispersion, were incorporated by Houston (1980) and the solution of Equation 11 was applied to the geometry of the experimental study of Hales (1980) (i.e., a uniform slope with a shore-connected breakwater perpendicular to the shoreline). A problem in simulating those experimental hydraulic tests numerically was that the waves broke in the experimental facility near the shoreline and thus dissipated their energy. No mechanism existed to dissipate energy in the numerical model. However, dissipation was simulated by allowing waves to continue to propagate out of the problem area. The breakwater and uniform slope were simulated only to the point where breaking occurred. The depth was then increased to the depth of a semi-infinite ocean region surrounding the region of computation, and the waves were allowed to radiate away from the area of interest.

34. Figure 11 shows a typical comparison between the experimental

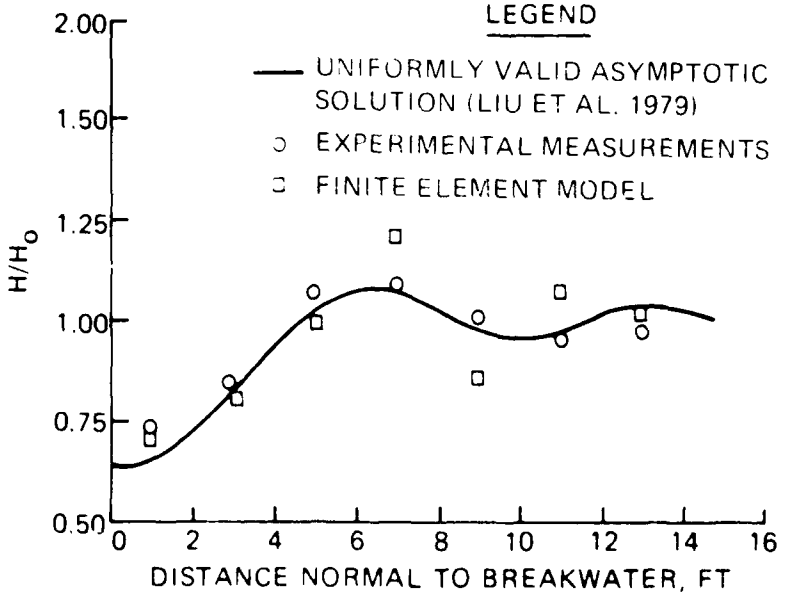


Figure 11. Comparison of asymptotic and finite element solutions with experimental measurements (breakwater normal to shoreline) (after Houston 1980)

measurements and the finite element calculations of [illegible] is the uniformly valid asymptotic solution derived by Liu and Lozano (1979). The solution derived by Liu and Lozano (1979) appears to be in good agreement with the experimental tests. The finite element calculations seem to also agree quite well in the shadow zone with the experimental tests. The agreement is not as good outside the shadow zone. The difference is partially attributed to the artificial increase in depth to allow the waves to radiate to the outer region. This depth transition would cause some energy to reflect back into the region of interest.

PART IV: UNIFORMLY VALID ASYMPTOTIC THEORY

Background

35. In recent years, the parabolic approximation has been developed extensively for studying wave scattering problems in different branches of the physical sciences. Radder (1979) and Lozano and Liu (1980) derived independently the parabolic approximation for water-wave problems. Analytical solutions were obtained for the combined refraction and diffraction wave field near a thin breakwater perpendicular to the shoreline on a plane beach. The background wave field was assumed to have straight-line wave rays.

36. Also based on the parabolic approximation, a numerical study of water-wave refraction and diffraction problems has been conducted by Tsay and Liu (1982), where the refraction index is not constant. Two problems were considered: (a) the wave field near a submerged shoal on a sloping bottom and (b) the wave field in the neighborhood of a breakwater on a sloping beach. In the latter problem, the orientation of the breakwater is no longer limited to be perpendicular to the shoreline. For the perpendicular case, the accuracy of the parabolic approximation numerical results was verified by comparing with the precise experimental data of Pantazaras (1979) and Hales (1980) (both sets of data having been obtained at WES). For this case, the uniformly valid asymptotic theory of Liu, Lozano, and Pantazaras (1979) was also used to verify the parabolic approximation.

Development of Asymptotic Theory

37. For the case of a shore-connected breakwater on a linear plane beach, Liu and Mei (1976) and Lozano and Liu (1980) have previously shown that an approximate closed form solution can be obtained by the parabolic approximation. However, Liu (1982) showed that this solution becomes invalid near the tip of the breakwater because of the inherent nature of the parabolic approximation. To remove this weakness, Liu, Lozano, and Pantazaras (1979) and Liu (1982) developed a a uniformly valid asymptotic solution for the same problem. This theory is verified by the experimental data of the present study. The beach topography, which is required to be uniform in the alongshore direction, can be arbitrary in the onshore-offshore direction.

38. Following the development of Liu, Lozano, and Pantazaras (1979), and in the notation of Liu (1982), the geometry of the breakwater is generalized to be one of radiated wave rays emitted from the tip of the breakwater (Figure 12). The relation between the incident wave angle and the reflected wave angle along the breakwater is shown in Figure 13. Small-amplitude incident

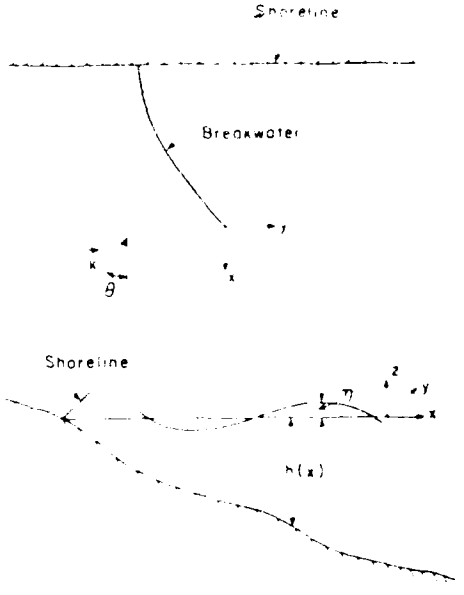


Figure 12. Definitive sketch, uniformly valid asymptotic theory (Liu, Lozano, and Pantazaras 1979)

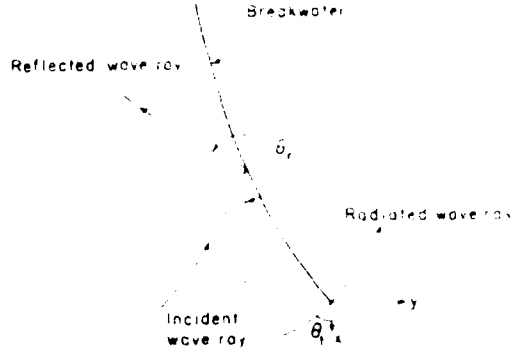


Figure 13. Definitive sketch, uniformly valid asymptotic theory, relation between incident wave angle and the reflected wave angle along the breakwater (Liu, Lozano, Pantazaras 1979)

waves with the incident wave amplitude, a_0, and radian frequency, ω, are assumed in the development. The angle of incidence is defined as θ_0. Liu, Lozano, and Pantazaras (1979) have shown that the leading order asymptotic solution for the free surface displacement, η, is

$$\eta(x, y, t) = \gamma A(x)\left[G(\theta)e^{iS} + G(\bar{\theta})e^{-iS}\right]e^{-i\omega t} \qquad (14a)$$

and the velocity potential can be expressed as:

$$\phi(x, y, z, t) = -\frac{ig\eta(x, y, t)}{\omega}\frac{\cosh k(z + h)}{\cosh kh} \qquad (14b)$$

where

$$\gamma = \pi^{-1/2}\exp\left(-\frac{i\pi}{4}\right) \qquad (14c)$$

and

$$G(\rho_1) = \int_{-\infty}^{\rho_1} e^{i\sigma^2}\, d\sigma \tag{15}$$

A(x) represents the combined refraction and shoaling factor:

$$A(x) = a_o \left[\left(\frac{\cos\theta_o}{\cos\theta}\right)\left(\frac{2k_o h_o + \sinh 2k_o H_o}{2kh + \sinh 2kh}\right)\right]^{1/2} \frac{\cosh kh}{\cosh k_o h_o} \tag{16}$$

The subscript o in Equation 16 denotes the quantities associated with incident waves in the far field. The function $G(\rho_1)$ defined in Equation 15 can be given in terms of the Fresnel integrals, whence:

$$G(\rho_1) = \frac{\pi^{1/2}}{8^{1/2}}(1 + i) + \frac{\pi^{1/2}}{2^{1/2}}\left[C_2\left(\rho_1^2\right) + iS_2\left(\rho_1^2\right)\right] \tag{17}$$

where

$$C_2\left(\rho_1^2\right) = \frac{1}{2\pi}\int_o^{\rho_1^2} \frac{\cos\tau}{\tau^{1/2}}\, d\tau \tag{18}$$

and

$$S_2\left(\rho_1^2\right) = \frac{1}{2\pi}\int_o^{\rho_1^2} \frac{\sin\tau}{\tau^{1/2}}\, d\tau \tag{19}$$

are the Fresnel cosine and sine integrals, respectively. The arguments of the function $G(\rho_1)$ in Equation 15 were defined by Liu (1982) as:

$$\sigma^2 = k - S \tag{20}$$

and

$$\bar{\sigma}^2 = k - \bar{S} \tag{21}$$

where

S = phase function of the incident wave, deg

$\bar{S}$ = phase function of the reflected wave from the breakwater, deg

R = phase function of the radiated waves generated by an oscillatory point source at the tip of the breakwater, deg

These phase functions can be calculated according to the ray theory; i.e., S, $\bar{S}$, and R can be evaluated by the integral

$$I = \int_{0}^{\bar{r}} \bar{k} \cdot d\bar{r} \tag{22}$$

where $\bar{k}$ is the wave number vector representing each wave field, respectively.

39. The zero phase lines for S, $\bar{S}$, and R intercept at the tip of the breakwater ($x = 0$, $y = 0$). From Equation 22, the value of the phase function (S, $\bar{S}$, or R) at any arbitrary point can be considered as the sum of the wave number component in the radial direction between the arbitrary point and the tip of the breakwater. The value of the phase function for radiated waves at any arbitrary point is always greater than or equal to S and $\bar{S}$. The branches of the multivalued functions θ and $\bar{\theta}$ are defined by Liu (1982) as follows: The value of θ is negative inside the shadow region defined according to the geometrical optics theory and is positive elsewhere. $\bar{\theta}$ is positive in the reflection region and is negative elsewhere. In the case where the breakwater has a curved shape and coincides with one of the radiated wave rays from the tip, the phase function for the reflected wave is:

$$\bar{S} = -\int_{0}^{x} k \cos\theta \, dx + \bar{k}_r y \tag{23}$$

where

$$\bar{k}_r = k_r \sin\theta_i \tag{24}$$

k_r and θ_i represent the wave number of reflected wave angle along the breakwater, respectively.

40. The analytical development of the tree-section displacement downcoast of a shore-connected breakwater, based on the uniformly valid asymptotic theory (Equation 12), was based on the fundamental assumption that the breakwater geometry in planform followed a radiated wave ray emitted from the tip of the breakwater. The detailed derivation of this and other expressions

required in the development are presented in Liu, Lozano, and Pantazaras (1979). However, in actuality, the planform layout of most shore-connected breakwaters at an angle to the shoreline is that of a straight line. Hence, while the assumption of a planform layout following a wave ray radiated from the tip of the structure expedited the analytical development, the numerical model based on this development does not precisely conform with most prototype conditions. It was therefore desirable to adapt the numerical scheme to fit the case of a straight breakwater at an angle to the shoreline. For the case of a straight breakwater, the phase function S_s for the incident wave, the phase function $\bar{S}_s$ for the reflected waves from the breakwater, and the phase function R_s for the radiated waves generated by the oscillatory point source at the tip of the breakwater can be expressed respectively as:

$$S_s = -\int_o^x k \cos\theta \, dx + \bar{K}_o y \tag{25}$$

$$\bar{S}_s = -\int_o^x k \cos\theta \, dx - \bar{K}_o y \tag{26}$$

$$R_s = -\int_o^x k \cos\theta \, dx + \bar{K}_t y \tag{27}$$

where

$$\bar{K}_o = k \sin\theta = k_o \sin\theta_o \tag{28}$$

$$\bar{K}_t = k \sin\theta = k_t \sin\theta_t \tag{29}$$

k_t and θ_t represent the wave number and the initial angle of incidence of a radiated wave ray, respectively. The numerical model developed under contract by Liu based on the uniformly valid asymptotic theory of Liu, Lozano, and Pantazaras (1979) was modified to conform to a straight breakwater at an angle to the shoreline, and is presented in Appendix B along with a sample output from the program. This numerical model served as the basis for comparison with the experimental results of the present study.

waves with truly deepwater characteristics. In addition, the requirement existed that the generated wave heights should be large enough so that small changes in the heights could be detectable, and at the same time the heights should be small enough so that comparisons with linear theories could be performed. Preliminary tests and experimental results of Hales (1980), indicated that for the area of major interest, and for the range of wave periods considered pertinent, the specific experimental conditions shown in Table 1 could be investigated with height changes remaining essentially linear, thus permitting comparisons with theoretical development.

43. Thirty-six wave-height sensors were used to determine the wave heights along the four sections parallel with the shoreline and downwave of the vertical breakwater (eight gages along two sections, nine gages along two sections, and two reference gages in the ocean region near the wave generator). The average of 90 wave heights recorded at each of the two ocean gages was considered to be the input wave height from the wave generator. The average of 90 wave heights recorded at the remaining 34 gages was normalized to this input wave height. All 36 gages were recorded simultaneously with each individual reading at all data stations consisting of the average of these 90 waves. Ten repeatability replication tests were conducted under identical test conditions to define the variability of the measurements (experimental scatter). The statistical measure of the variability was the square root of the variance, or the standard deviation. The variance is defined as the sum of the squares of the deviates of each individual observation from their average, divided by one less than the total number of deviates. One standard deviation was usually only 1 to 3 percent of the average value of the observations.

44. The wave-height data were obtained with the wave sensors positioned at 2-ft intervals starting at the breakwater and extending along the sections parallel with the shoreline downwave from the breakwater. To provide better definition of the wave field, the gages were offset a distance of 1 ft along the sections parallel with the shoreline and hence were repositioned at 2-ft intervals along the same sections. This provided information at 1-ft intervals for a distance of 16 ft from the breakwater along two sections and for a distance of 18 ft from the breakwater along the remaining two sections.

Test Facility

45. The irregular wave tests were conducted in the experimental facility

were sufficiently small such that the waves were nearly linear within the measurement region shown in Figure 1. The degree of linearity is displayed in Tables 2-19 for representative replications of each different wave characteristic (period and height) tested. It was found that at the ocean gages, on the average, 97.32 percent of the energy of the wave form exists at the generated period (fundamental frequency). Of the remaining 2.68 percent of the energy, 2.39 percent exists at the first harmonic. The distribution of the wave energy throughout the model for these representative experiments is also displayed in Tables 2-19. Here is seen the manner in which nonlinear effects are detectable as the wave propagates shoreward past the various gages. A portion of the energy of the wave form is redistributed from the fundamental frequency to higher harmonics, and along the section at a distance of 6 ft from the shoreline only 89.05 percent exists (on the average) at the generated period. (This percentage increases with distance deep into the shadow zone protected by the breakwater.) However, of the remaining 10.95 percent of the energy, 8.32 percent exists (on the average) at the first harmonic.

46. The deviation from linearity, and hence the acceptability of comparisons of these experimental data with theoretical studies based on the small amplitude assumption, can be investigated by observing the rapidity with which the wave energy redistributes from the fundamental frequency as the waves shoal shoreward. For constant values of wave period T of 0.75 sec, 1.00 sec, and 1.50 sec, Figures 14-16, respectively, display the percentage of total energy in the fundamental period with distance from the shoreline, for three initial values of wave height each. In all cases, the nonlinearities are apparent as the waves propagate into shallower water with the least alteration occurring to the waves with the smaller period and the smaller initial generated wave height. As the wave height increases, the effects of the nonlinearity become substantially greater for constant values of wave period. At the same time, the longer period waves appear to exhibit at least as great [illegible] from the fundamental [illegible] This is attributed in large part to reflection of long-period waves from the breakwater, and to a lesser degree, to resonance within the experimental basin [illegible]. The asymptotic theory used as a basis for comparison of these experimental data provides for a specific wave period analysis, although the [illegible] assumptions regarding wave height and flood portion of dynamics are adhered to

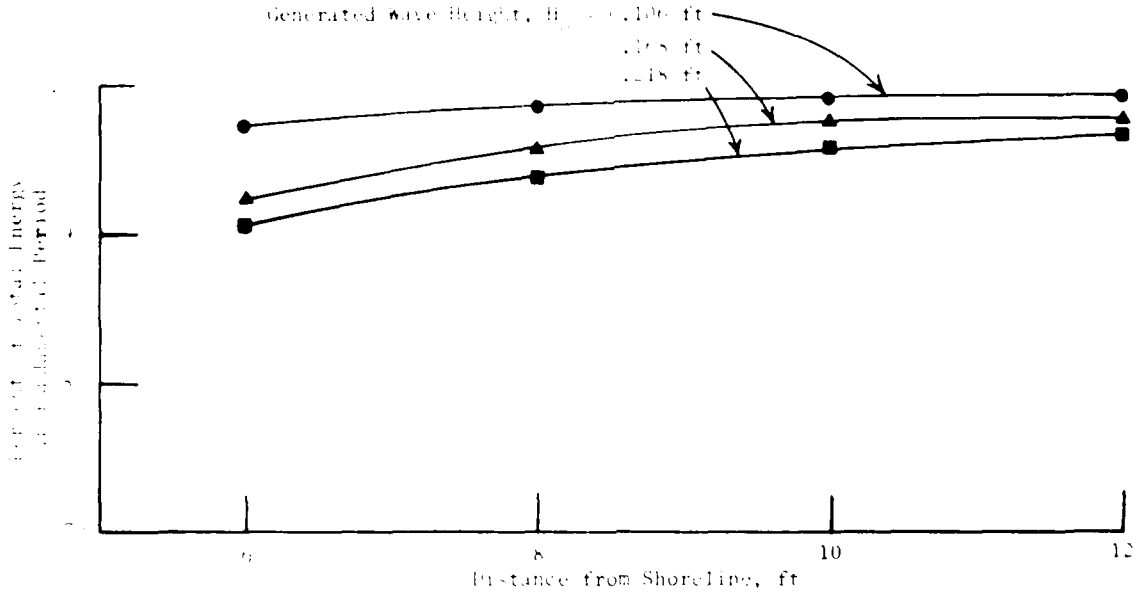


Figure 14. Effects of nonlinearities (initial wave height and shoaling effects) on percent of total energy in fundamental period, wave period = 0.75 sec

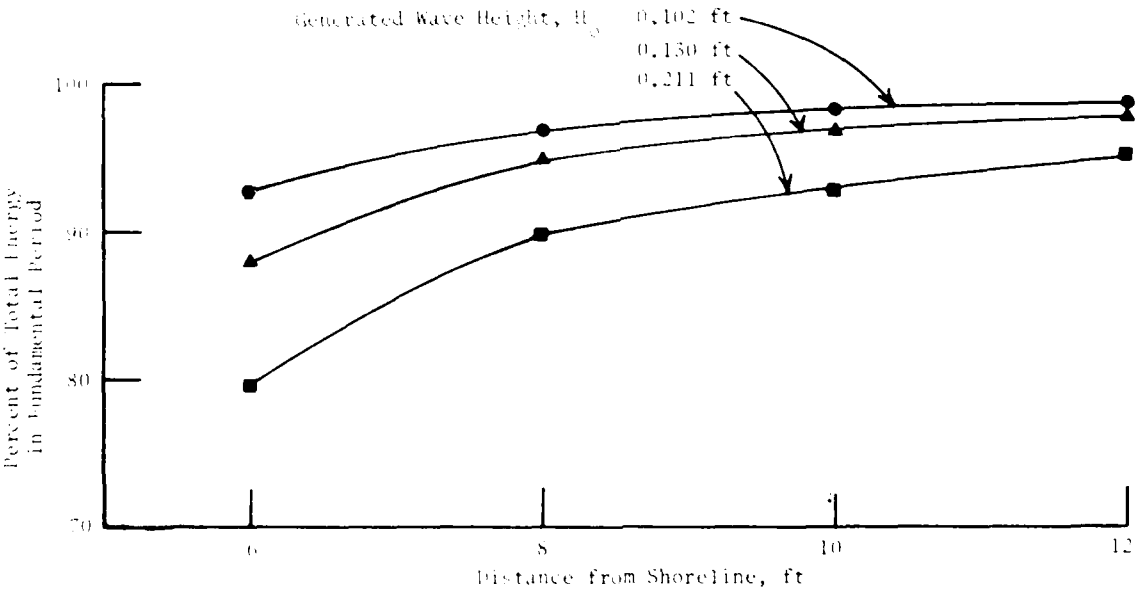


Figure 15. Effects of nonlinearities (initial wave height and shoaling effects) on percent of total energy in fundamental period, wave period = 1.00 sec

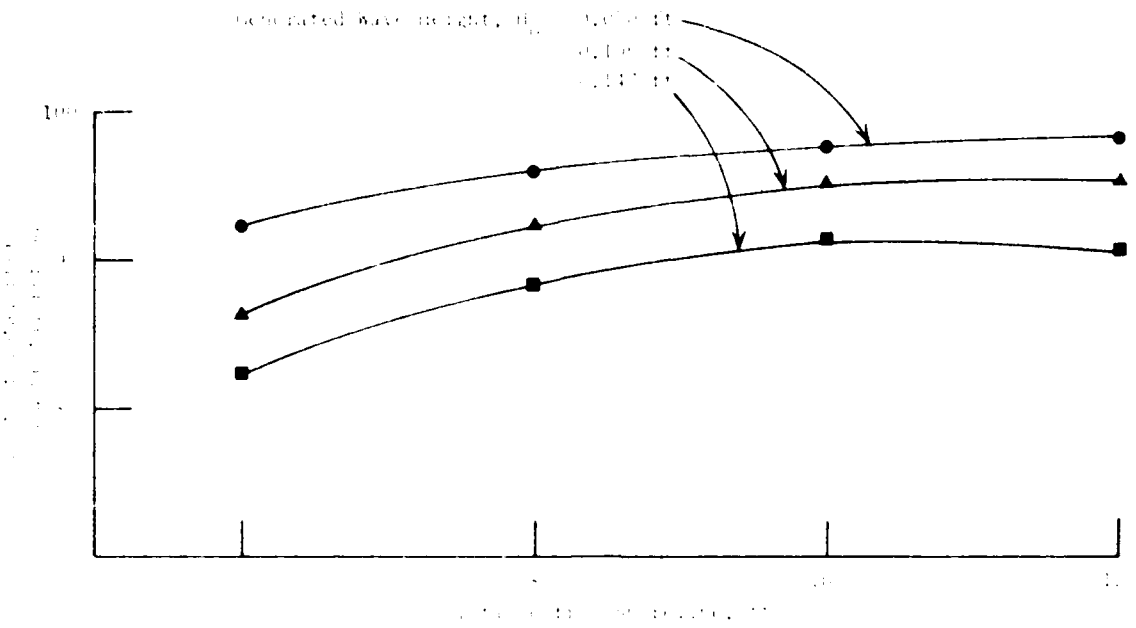


Figure 16. Effects of nonlinearities (initial wave height and shoaling effects) on percent of total energy in fundamental period, wave period = 1.50 sec

Wave-Height Data

47. Wave patterns covering the conditions tested in this experimental study are shown in Photos 1-9. Wave-height amplification coefficients, H/H_o, for all conditions tested are shown in Plates 1-36. Each of these plates constitutes a section parallel with the shoreline at distances from the shoreline of 6, 8, 10, or 12 ft. The corresponding scatter of the experimental data is indicated by the error bars of plus-or-minus one standard deviation. Also shown in these plates are the wave-height amplification due to diffraction alone (as if the basin were horizontal beyond the structure) and the wave-height amplification as indicated by the uniformly valid asymptotic theory.

Study Results

48. For small wave periods, small initial wave heights, and deep within the shadow zone (sections near the shoreline), both the diffraction theory and the uniformly valid asymptotic theory predict wave-height amplifications that

compare consistently well with the experimental data. Nearer the tip of the breakwater the diffraction theory tends to diverge rapidly from the experimental results, and the uniformly valid asymptotic theory more nearly approximates the experiments. As wave period increases, the deviation of the diffraction theory becomes more pronounced at all locations except very near the breakwater; however, the uniformly valid asymptotic theory continues to closely approximate the data from the experiments in all regions. The greatest deviation of the uniformly valid asymptotic theory from the experiments occurs outside the breakwater shadow zone in the area of the asymptotic undulations of the wave-height amplification factor. This can be attributed to reflection of longer period wave energy from the experimental beach breaker zone not accounted for in the analytical development, since the uniformly valid asymptotic theory consistently underpredicts the wave-height amplification in this area.

49. In general, it can be concluded that the uniformly valid asymptotic theory is superior to diffraction theory alone for estimating wave heights downcoast of nearshore structures subjected to combined refraction and diffraction. Particularly for longer period waves and for the region near the structures where scour and erosion are known to frequently occur, this theory provides an estimation that consistently approximates the results of this experimental study and that is significantly better than diffraction theory. At the same time, it appears desirable to incorporate into this theory a degree of nonlinearity that is not presently available.

50. Because the uniformly valid asymptotic theory compares favorably with the results of this experimental study (four sections parallel with the shoreline for three wave periods with three incident wave heights each), the numerical program developed from this theory and modified to fit the straight breakwater geometry was applied to the entire shadow zone region of the structure. These results are presented in Plates 37-50 for distances parallel to shoreline of 1 ft through 14 ft, respectively, in increments of 1-ft displacement seaward toward the tip of the breakwater (which was located 15 ft from the shoreline). The three wave periods utilized in the experimental phase of this study (0.75, 1.00, and 1.50 sec) are shown in these plates as the independent parameters. It is apparent near the shoreline that the longer period waves allow for the greater wave-height amplification at all locations (particularly within the shadow zone). With increase in distance from the

shoreline, the longer period waves continue to exhibit a greater wave-height amplification in the shadow zone; however, outside the shadow zone, the period effect becomes less apparent and eventually becomes entirely obscured as the undulations of the different periods engulf (overlap) each other.

PART VI: CONCLUSIONS

51. Based on results from the three-dimensional experimental investigation, reported herein, comparisons with existing diffraction theory, and a new uniformly valid asymptotic theory reported herein, it is concluded that:

a. For short wave periods, small initial wave heights (linear waves), and locations well within the shadow zone of the structure, both the diffraction theory and the uniformly valid asymptotic theory predict wave-height amplifications that compare consistently well with the experimental data. However, nearer the tip of the structure, diffraction theory tends to diverge rapidly from the experimental results, and the uniformly valid asymptotic theory more nearly approximates the experiments.

b. As the wave period increases, the deviation of the diffraction theory becomes more pronounced at all locations except very near the structure; however, the uniformly valid asymptotic theory continues to closely approximate the data from the experiments in all regions.

c. The uniformly valid asymptotic theory is superior to diffraction theory for estimating wave heights downcoast of nearshore structures subjected to combined refraction and diffraction. Particularly for longer period waves and for the region near the structures where scour and erosion are known to occur, this theory consistently approximates the results of this experimental study.

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Table 1

Experimental Conditions Tested

Initial Wave Heights, H_o (ft) near Wave Generator		
Wave Period, T (sec)		
0.75	1.00	1.50
0.106	0.102	0.070
0.168	0.130	0.106
0.218	0.211	0.147

Note: All experimental test conditions were replicated 10 times.

Table 3

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 0.75 sec, Ocean Wave Height = 0.106 ft

60-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 0.75 sec	First Harmonic 0.38 sec	Second Harmonic 0.25 sec	Third Harmonic 0.19 sec	Fourth Harmonic 0.15 sec
1*	98.42(+99)	1.55(+161)	0.00(+155)	0.02(+76)	0.01(+86)
2*	98.61(+144)	1.30(-60)	0.09(+90)	0.00(+159)	0.00(+76)
3	96.63(-149)	0.35(-170)	0.02(+147)	0.00(-113)	0.00(-131)
4	99.8[illegible](-127)	0.06(+126)	0.04(-112)	0.01(-116)	0.01(-106)
5	98.93(-16)	1.01(-39)	0.04(-48)	0.01(-67)	0.00(-61)
6	99.35(+101)	0.60(-179)	0.02(+43)	0.02(+88)	0.01(+81)
7	99.70(-134)	0.26(+99)	0.02(-108)	0.01(-131)	0.01(-118)
8	90.01(-28)	0.95(-55)	0.03(-67)	0.01(-83)	0.00(-81)
9	99.30(+91)	0.66(+156)	0.01(+84)	0.02(+72)	0.01(+73)
10	99.57(-147)	0.40(+68)	0.03(-93)	0.01(-135)	0.00(-128)
11	99.64(-173)	0.35(-118)	0.01(+126)	0.00(-169)	0.00(-143)
12	99.68(+128)	0.27(+94)	0.02(+143)	0.02(+73)	0.01(+90)
13	99.81(+140)	0.17(-135)	0.00(+118)	0.01(+77)	0.00(+119)
14	99.86(-132)	0.10(+141)	0.02(-102)	0.01(-115)	0.01(-118)
15	98.53(-11)	1.39(-42)	0.07(-43)	0.01(-59)	0.00(-52)
16	99.05(+114)	0.92(-153)	0.01(+18)	0.02(+92)	0.01(+85)
17	99.23(-131)	0.67(+104	0.04(-78)	0.01(-127)	0.01(-122)
18	98.46(-29)	1.44(-54)	0.08(-71)	0.02(-83)	0.01(-86)
19	99.19(+109)	0.69(+96)	0.08(+95)	0.02(+87)	0.02(+78)
20	99.84(+56)	0.04(+27)	0.08(+29)	0.04(+64)	0.01(+73)
21	99.35(+59)	0.57(+72)	0.04(+52)	0.03(+69)	0.01(+85)
22	99.63(+136)	0.33(-81)	0.03(+74)	0.01(+77)	0.00(+87)
23	98.73(-123)	1.17(+121)	0.03(-72)	0.02(-120)	0.01(-125)
24	96.77(-9)	3.05(-18)	0.15(-24)	0.03(-24)	0.00(-35)
25	97.45(+124)	2.47(-126)	0.07(+12)	0.01(+127)	0.00(+81)
26	98.45(-120)	1.46(+124)	0.05(-46)	0.03(-115)	0.01(-123)
27	97.22(-10)	2.62(-26)	0.13(-43)	0.02(-56)	0.01(-60)
28	99.85(+146)	0.13(-134)	0.01(-171)	0.02(+88)	0.01(+97)
29	99.[illegible](+59)	0.25(+37)	0.07(+48)	0.04(+68)	0.02(+70)
[illegible]	[illegible]	0.1[illegible](+64)	0.01(-101)	0.00(-73)	0.00(-24)
31	99.6[illegible](+19)	0.3[illegible](+11)	0.01(-21)	0.01(-9)	0.00(+2)
32	97.49(+77)	2.42(+123)	0.04(+129)	0.03(+10)	0.02(+8)
33	98.42(+171)	1.46(-44)	0.02(+124)	0.00(+15)	0.00(+102)
34	95.20(-85)	4.71(-169)	0.03(+125)	0.01(-58)	0.01(-107)
35	92.2[illegible](+43)	7.03(+81)	0.60(+113)	0.04(+124)	0.01(+75)
36	94.1[illegible](+171)	5.5[illegible](-30)	0.35(+135)	0.02(-40)	0.00(+131)

* Ocean gage.

Table 4

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 0.75 sec, Ocean Wave Height = 0.168 ft

60-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 1

Gage	Fundamental Period 0.75 sec	First Harmonic 0.38 sec	Second Harmonic 0.25 sec	Third Harmonic 0.19 sec	Fourth Harmonic 0.15 sec
1*	93.66(+90)	6.00(+170)	0.29(-103)	0.05(+77)	0.00(+170)
2*	97.47(+141)	1.90(+11)	0.58(-173)	0.04(+12)	0.00(+156)
3	98.68(-122)	1.26(-44)	0.01(-119)	0.05(-125)	0.01(-116)
4	99.39(-160)	0.08(+129)	0.03(+126)	0.00(-126)	0.01(-118)
5	97.88(-92)	2.09(-158)	0.02(+178)	0.00(-140)	0.00(-124)
6	96.58(+30)	3.26(+99)	0.15(+86)	0.00(+97)	0.01(+52)
7	97.29(+144)	2.53(-74)	0.17(+82)	0.01(-107)	0.00(+147)
8	97.33(-93)	2.61(-178)	0.06(+117)	0.00(-74)	0.00(-112)
9	96.33(+18)	3.45(+33)	0.20(+56)	0.03(+80)	0.01(+60)
10	97.20(+134)	2.69(-105)	0.10(+39)	0.01(-173)	0.00(+114)
11	99.06(+146)	0.89(-178)	0.05(-122)	0.01(+35)	0.00(+132)
12	99.91(+119)	0.01(+134)	0.05(+165)	0.02(+12)	0.01(+110)
13	98.47(+168)	1.47(-42)	0.06(+143)	0.00(-96)	0.00(-132)
14	97.64(-94)	2.33(-175)	0.02(+174)	0.01(-50)	0.00(-120)
15	95.40(+21)	4.20(+43)	0.37(+76)	0.03(+80)	0.00(+43)
16	96.11(+154)	3.70(-59)	0.19(+101)	0.01(-86)	0.00(-108)
17	96.33(-102)	3.43(+166)	0.12(+55)	0.01(-48)	0.00(-90)
18	94.90(+15)	4.72(+26)	0.35(+56)	0.03(+73)	0.00(+34)
19	98.83(+162)	1.05(-22)	0.12(+90)	0.01(-179)	0.00(-6)
20	99.06(+77)	0.88(+83)	0.06(+58)	0.01(+81)	0.00(+73)
21	99.30(+41)	0.06(+79)	0.01(-3)	0.02(+58)	0.01(+63)
22	98.24(+86)	1.72(+123)	0.02(+176)	0.02(+58)	0.01(+74)
23	97.01(+173)	2.84(-15)	0.14(+157)	0.01(+7)	0.00(-129)
24	94.97(-93)	4.87(+178)	0.15(+89)	0.01(-49)	0.00(-122)
25	90.42(+34)	9.56(+69)	0.94(+53)	0.05(+116)	0.03(+39)
26	92.8 (+169)	6.71(-49)	0.60(+113)	0.05(-65)	0.03(+33)
27	95.90(-82)	3.99(-168)	0.21(+135)	0.00(+39)	0.01(-117)
28	97.1 (+177)	2.8 (+167)	0.03(+133)	0.00(+85)	0.01(+60)
29	99.2 (+13)	0.7 (+39)	0.04(-57)	0.02(+36)	0.01(+56)
30	98.3 (+8)	1.5 (+ 9)	0.00(-133)	0.00(139)	0.00(-34)
31	98.9 (+11)	1.0 (+73)	0.05(+63)	0.01(+78)	0.00(+33)
32	96.1 (+113)	3.7 (-137)	0.23(-28)	0.04(+113)	0.01(+132)
33	91.5 (-105)	7.9 (+64)	0.79(-125)	0.03(+89)	0.01(-121)
34	93.7 (-101)	5.1 (-105)	2.03(-149)	0.16(-176)	0.01(+169)
35	90.83(+69)	7.0 (+123)	2.72(-190)	0.33(-124)	0.04(-73)
36	94.7 (-1)	3.1 (+39)	2.06(-140)	0.10(+35)	0.04(-154)

* Ocean gage.

Table 5

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 0.75 sec, Ocean Wave Height = 0.168 ft

90-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 0.75 sec	First Harmonic 0.38 sec	Second Harmonic 0.25 sec	Third Harmonic 0.19 sec	Fourth Harmonic 0.15 sec
1*	94.84(+153)	4.90(+161)	0.40(-94)	0.12(+49)	0.03(+108)
2*	95.18(+146)	4.13(-2)	0.68(+138)	0.00(-31)	0.01(+102)
3	99.70(-160)	0.27(-102)	0.01(-168)	0.01(-110)	0.00(-162)
4	97.30(-137)	1.96(+92)	0.13(-45)	0.02(-124)	0.00(-125)
5	97.56(-23)	2.13(-55)	0.12(-74)	0.03(-67)	0.01(-75)
6	97.30(+99)	2.52(+162)	0.02(-92)	0.05(+83)	0.01(+80)
7	98.36(-142)	1.55(+78)	0.08(-75)	0.01(-131)	0.00(-128)
8	97.19(-38)	2.62(-73)	0.13(-91)	0.05(-96)	0.01(-93)
9	97.31(+80)	2.52(+154)	0.04(-168)	0.02(+54)	0.01(+81)
10	97.56(-166)	2.33(+34)	0.07(-123)	0.00(+143)	0.00(+140)
11	98.47(-177)	1.42(-3)	0.10(+26)	0.01(-144)	0.00(+49)
12	99.16(+124)	0.73(+150)	0.07(-152)	0.01(+36)	0.01(+76)
13	99.29(+134)	0.67(-120)	0.02(+48)	0.02(+113)	0.00(+83)
14	99.33(-142)	0.64(+104)	0.01(-9)	0.01(-144)	0.00(-173)
15	95.83(-36)	3.86(-66)	0.26(-63)	0.05(-102)	0.01(-75)
16	96.37(+103)	3.24(-171)	0.10(-47)	0.07(+82)	0.01(+79)
17	97.28(-142)	2.57(+69)	0.14(-36)	0.01(-146)	0.00(-134)
18	95.79(-36)	3.91(-69)	0.23(-92)	0.05(-91)	0.01(-92)
19	99.71(+110)	1.23(-137)	0.01(+115)	0.00(-16)	0.01(+77)
20	97.26(+55)	2.62(+52)	0.03(+59)	0.05(+52)	0.02(+74)
21	95.21(+53)	4.61(+68)	0.12(+65)	0.04(+68)	0.02(+68)
22	97.93(+129)	1.92(-91)	0.15(+51)	0.00(+144)	0.00(+67)
23	96.63(-128)	3.13(+35)	0.21(-59)	0.01(-144)	0.00(-167)
24	92.61(-21)	6.64(-43)	0.69(-57)	0.05(-58)	0.01(-77)
25	92.86(+104)	6.93(-159)	0.37(-36)	0.05(+73)	0.01(+101)
26	97.12(-136)	2.43(+95)	0.35(-53)	0.04(-124)	0.00(-114)
27	90.73(-26)	8.51(-57)	0.53(-75)	0.07(-85)	0.01(-99)
28	98.79(+162)	1.13(+69)	0.00(-76)	0.01(+162)	0.00(+102)
29	92.06(+63)	7.41(+99)	0.07(+54)	0.05(+88)	0.01(+110)
30	94.13(+49)	5.76(-24)	0.04(-103)	0.01(-110)	0.00(-102)
31	97.23(-25)	2.61(+43)	0.12(-25)	0.02(-13)	0.00(-19)
32	94.61(+53)	5.24(+131)	0.08(-169)	0.00(+60)	0.01(+81)
33	90.92(+153)	8.42(-54)	0.57(+115)	0.04(-68)	0.00(+141)
34	90.40(-99)	8.79(+159)	0.73(+52)	0.06(-46)	0.01(-128)
35	85.13(+53)	13.27(+43)	1.37(+43)	0.47(+51)	0.14(+52)
36	88.61(+103)	10.08(+43)	1.27(+48)	0.24(+176)	0.03(-70)

* Ocean side.

Table 4

Distribution [illegible] throughout the [illegible] Facility, 30-deg

Incident [illegible] Weight = 0.218 lb

[illegible]

Pers [illegible] [illegible] Measurement No. 1

Gage	[illegible]	[illegible]	[illegible]	Second Harmonic (0.2) sec	[illegible]	Fourth Harmonic (0.1) sec
1*	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02(+77)
2*	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00(+96)
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.01(-113)
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00(+168)
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00(-145)
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02(-84)
7	[illegible]	[illegible]	[illegible]	[illegible]	.02(+72)	0.01(+59)
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.03(-131)
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02(-93)
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.01(+78)
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.31(+76)
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.01(+65)
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.03(+96)
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.08(-35)
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.34(-100)
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.01(+63)
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00(-153)
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02(-107)
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.09(+101)
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00(+24)
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.09(-49)
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02(+64)
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.01(+112)
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.01(-119)
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.05(-124)
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0[illegible](+5)
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00(-110)
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02(+71)
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00(-80)
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00(-54)
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.03(+41)
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.03(+104)
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.01(+167)
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.04(-21)
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.10(-98)
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.16(+109)

[illegible]

Table 7

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 0.75 sec, Ocean Wave Height = 0.218 ft

60-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 0.75 sec	First Harmonic 0.38 sec	Second Harmonic 0.25 sec	Third Harmonic 0.19 sec	Fourth Harmonic 0.15 sec
1*	94.49(+93)	4.86(+147)	0.61(-138)	0.03(-51)	0.02(+24)
2*	98.95(+143)	0.84(-7)	0.20(+176)	0.01(+60)	0.00(+113)
3	98.60(+162)	1.26(+75)	0.12(-149)	0.02(+47)	0.00(+146)
4	99.37(-172)	0.53(-29)	0.09(+128)	0.01(-30)	0.00(+100)
5	95.55(-73)	4.25(-134)	0.16(-179)	0.02(-161)	0.02(-99)
6	95.48(+38)	4.32(+76)	0.18(+117)	0.01(+85)	0.01(+83)
7	96.69(+158)	3.12(-54)	0.19(+127)	0.00(-17)	0.00(+39)
8	97.19(-98)	2.76(+175)	0.05(+118)	0.00(-120)	0.01(-134)
9	96.34(+20)	3.35(+41)	0.28(+56)	0.02(+78)	0.01(+53)
10	96.50(+131)	3.32(-95)	0.15(+57)	0.00(-171)	0.00(+174)
11	98.13(147)	1.83(-138)	0.04(-17)	0.01(+64)	0.01(+179)
12	99.52(+35)	0.32(+13)	0.14(+86)	0.01(+86)	0.01(+76)
13	99.09(+97)	0.20(+155)	0.09(+108)	0.01(+77)	0.01(+76)
14	96.33(+170)	3.43(-25)	0.24(+139)	0.01(-5)	0.00(+123)
15	94.48(-93)	5.31(-170)	0.19(+120)	0.00(+13)	0.01(-137)
16	92.46(+38)	6.81(+69)	0.63(+91)	0.07(+107)	0.03(+77)
17	94.57(+146)	5.12(-75)	0.31(+73)	0.00(-118)	0.00(-42)
18	95.25(-102)	4.59(+165)	0.15(+70)	0.01(-60)	0.00(-147)
19	97.71(+76)	2.13(+95)	0.11(+89)	0.04(+93)	0.01(+92)
20	99.99(+17)	0.01(-140)	0.00(-120)	0.00(+46)	0.00(+26)
21	99.62(+21)	0.36(+35)	0.00(-10)	0.01(+40)	0.00(+38)
22	95.75(+81)	4.05(+137)	0.15(+157)	0.03(+77)	0.01(+79)
23	92.23(+156)	7.23(-56)	0.54(+100)	0.01(-91)	0.00(+48)
24	90.02(-92)	9.59(+176)	0.36(+84)	0.02(-56)	0.01(-129)
25	84.85(+27)	12.65(+46)	2.06(+64)	0.33(+76)	0.10(+74)
26	89.60(+153)	9.35(-61)	0.99(+94)	0.06(-129)	0.01(+7)
27	91.80(-103)	7.94(+160)	0.23(+60)	0.03(-64)	0.01(-142)
28	98.06(+125)	1.92(-103)	0.02(+89)	0.00(+13)	0.00(+39)
29	98.77(+35)	1.18(+0.9)	0.03(-23)	0.01(-33)	0.01(+3)
30	99.67(-27)	0.32(-31)	0.01(+54)	0.01(-85)	0.00(-62)
31	98.77(-12)	1.19(-31)	0.04(+30)	0.01(-11)	0.00(-4)
32	91.39(+25)	7.80(+42)	0.70(+55)	0.10(+69)	0.02(+60)
33	87.77(+82)	11.39(+16)	0.78(-135)	0.03(-8)	0.03(+45)
34	83.33(+175)	14.65(-20)	1.78(+143)	0.22(-43)	0.03(+108)
35	81.49(-76)	16.60(-164)	1.75(+111)	0.12(-10)	0.04(-122)
36	80.89(+52)	15.36(+86)	3.01(+108)	0.59(+114)	0.15(+117)

* Ocean gage.

Table 8

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 1.00 sec, Ocean Wave Height = 0.102 ft

60-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 1

Gage	Fundamental Period 1.00 sec	First Harmonic 0.50 sec	Second Harmonic 0.33 sec	Third Harmonic 0.25 sec	Fourth Harmonic 0.20 sec
1*	99.19(+86)	0.65(+116)	0.16(+174)	0.01(-133)	0.00(-25)
2*	99.44(+122)	0.53(-96)	0.00(-125)	0.02(+13)	0.01(+119)
3	99.64(+163)	0.32(+156)	0.04(-56)	0.00(+128)	0.00(+27)
4	99.60(+137)	0.33(+147)	0.05(-141)	0.02(+10)	0.01(+113)
5	98.07(+176)	1.91(-13)	0.01(+146)	0.00(-147)	0.00(+51)
6	98.40(-103)	1.57(+165)	0.01(+74)	0.00(-116)	0.00(-130)
7	98.93(-25)	1.07(-58)	0.01(-72)	0.00(-171)	0.00(+38)
8	98.23(+57)	1.67(+93)	0.03(+111)	0.01(+101)	0.00(+116)
9	98.75(+125)	1.23(-119)	0.01(+54)	0.00(+68)	0.00(+106)
10	99.16(-156)	0.81(+32)	0.02(-136)	0.00(+99)	0.00(-136)
11	99.13(-15)	0.84(-22)	0.01(-60)	0.02(+142)	0.01(+142)
12	98.13(-27)	1.84(-50)	0.03(-98)	0.00(+105)	0.00(+65)
13	99.50(-6)	0.49(+19)	0.00(-169)	0.01(+179)	0.00(-142)
14	98.33(+64)	1.63(+105)	0.04(+106)	0.00(+140)	0.00(+79)
15	97.98(+144)	1.95(-84)	0.07(+54)	0.00(+156)	0.00(+61)
16	97.41(-130)	2.51(+94)	0.08(-49)	0.00(-150)	0.00(-144)
17	98.44(-63)	1.54(-121)	0.01(-159)	0.00(-67)	0.00(-77)
18	98.25(+18)	1.72(+30)	0.03(+35)	0.00(-0.5)	0.00(+22)
19	99.76(-115)	0.16(-41)	0.07(+177)	0.01(+151)	0.00(-82)
20	99.93(-171)	0.02(+125)	0.04(+16)	0.01(-131)	0.00(+94)
21	99.91(+170)	0.05(+8)	0.02(-18)	0.01(+146)	0.00(-15)
22	98.95(-166)	1.03(+1)	0.01(+54)	0.01(-106)	0.00(+78)
23	97.02(-109)	2.93(+133)	0.05(+7)	0.01(-126)	0.00(+127)
24	96.26(-40)	3.63(-88)	0.11(-113)	0.00(-109)	0.00(-141)
25	92.83(+46)	6.81(+79)	0.50(+118)	0.04(+133)	0.00(+152)
26	93.62(+124)	5.99(-128)	0.35(-6)	0.04(+113)	0.00(-93)
27	96.46(-153)	3.37(+36)	0.16(-125)	0.01(+83)	0.00(-91)
28	98.97(+93)	0.96(+162)	0.06(+140)	0.01(-165)	0.00(-17)
29	97.30(+63)	2.12(+33)	0.04(+13	0.03(+62)	0.01(+98)
30	97.27(+42)	2.67(+47)	0.02(+35)	0.04(-6)	0.01(+24)
31	95.61(+59)	4.25(103)	0.11(+123)	0.02(+100)	0.02(+102)
32	94.88(+105)	4.91(-165)	0.20(-79)	0.01(-6)	0.00(+59)
33	90.94(+172)	8.23(-36)	0.63(+120)	0.08(-77)	0.02(+108)
34	85.08(-110)	12.34(+124)	1.82(+2)	0.24(-112)	0.03(+154)
35	83.24(-31)	13.73(-82)	2.53(-121)	0.44(-153)	0.06(-168)
36	88.32(+55)	10.19(+90)	1.34(+123)	0.15(+159)	0.00(-164)

* Ocean gage.

Table 9

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 1.00 sec, Ocean Wave Height = 0.102 ft

60-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 1.00 sec	First Harmonic 0.50 sec	Second Harmonic 0.33 sec	Third Harmonic 0.25 sec	Fourth Harmonic 0.20 sec
1*	99.10(+80)	0.75(+112)	0.13(-141)	0.01(-81)	0.00(+128)
2*	99.54(+118)	0.39(-77)	0.03(-29)	0.03(-38)	0.01(+76)
3	99.23(+147)	0.73(+114)	0.03(-140)	0.01(+111)	0.00(-61)
4	99.28(+159)	0.71(-101)	0.01(+23)	0.00(-178)	0.00(-117)
5	98.21(-135)	1.75(+81)	0.04(-38)	0.00(+56)	0.00(-49)
6	98.41(-53)	1.53(-94)	0.05(-120)	0.00(+156)	0.00(-175)
7	98.09(+29)	1.83(+33)	0.03(+37)	0.00(+71)	0.00(+138)
8	98.02(+104)	1.96(-148)	0.02(-41)	0.00(+93)	0.00(+137)
9	99.00(+176)	0.99(-17)	0.01(+172)	0.00(-100)	0.00(+42)
10	98.37(-98)	1.11(+156)	0.01(+32)	0.00(-115)	0.00(-163)
11	98.39(+20)	1.47(+68)	0.10(-101)	0.03(-53)	0.01(-24)
12	97.92(-20)	2.06(-37)	0.02(+51)	0.00(+143)	0.01(+152)
13	98.57(-11)	1.37(+9)	0.02(-154)	0.03(-161)	0.01(-175)
14	99.65(+34)	0.35(+31)	0.00(+108)	0.00(-61)	0.00(-153)
15	98.41(+115)	1.54(-143)	0.03(-69)	0.02(+31)	0.00(+12)
16	96.26(-153)	3.58(+38)	0.20(-136)	0.01(+37)	0.00(-2)
17	97.20(-79)	2.74(-171)	0.05(+103)	0.00(-50)	0.00(-2)
18	97.16(-4)	2.71(-22)	0.11(-41)	0.01(-68)	0.00(-54)
19	99.50(-133)	0.45(-51)	0.02(+90)	0.03(-32)	0.01(-174)
20	99.87(-166)	0.01(+174)	0.07(+75)	0.03(-118)	0.01(+92)
21	99.66(-122)	0.29(-51)	0.01(+125)	0.03(+177)	0.01(+51)
22	97.48(-126)	2.44(+80)	0.07(-53)	0.00(+88)	0.00(-105)
23	96.08(-59)	3.80(-123)	0.11(-177)	0.00(+109)	0.00(-14)
24	93.64(+17)	5.96(+23)	0.36(+36)	0.03(+55)	0.01(+69)
25	93.08(+102)	6.42(-172)	0.50(-78)	0.05(+26)	0.00(+153)
26	94.34(-138)	5.23(-5)	0.34(-176)	0.03(-5)	0.01(-148)
27	95.59(-97)	4.23(+154)	0.11(+42)	0.01(-89)	0.00(-154)
28	94.91(+122)	4.89(-34)	0.13(-84)	0.06(+82)	0.00(-2)
29	93.32(+44)	6.44(+150)	0.17(+176)	0.02(+132)	0.00(-35)
30	93.25(+98)	6.58(+33)	0.09(+13)	0.05(+31)	0.01(+104)
31	96.17(+48)	3.40(+99)	0.41(+97)	0.03(+119)	0.02(+101)
32	95.97(+43)	3.89(+176)	0.13(-137)	0.00(-36)	0.00(+3)
33	92.66(+150)	6.87(-59)	0.41(+83)	0.05(-157)	0.01(+100)
34	86.91(-130)	11.24(+79)	1.69(-63)	0.24(+168)	0.02(+44)
35	82.93(-51)	14.31(-120)	2.32(-130)	0.38(+125)	0.05(+86)
36	83.31(+31)	13.76(+56)	2.05(+56)	0.34(+91)	0.05(+129)

* Ocean gage.

Table 10

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 1.00 sec, Ocean Wave Height = 0.130 ft

60-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 1

Gage	Fundamental Period 1.00 sec	First Harmonic .50 sec	Second Harmonic 0.33 sec	Third Harmonic 0.25 sec	Fourth Harmonic 0.20 sec
1*	99.22(+95)	0.59(+107)	0.13(+128)	0.01(+149)	0.00(+63)
2*	99.09(123)	0.89(-102)	0.01(-32)	0.01(+75)	0.00(-141)
3	99.82(+169)	0.15(-172)	0.01(-33)	0.02(+118)	0.00(-121)
4	99.91(+142)	0.07(-136)	0.01(+162)	0.04(+46)	0.00(+119)
5	99.44(-177)	0.55(+6)	0.00(-41)	0.00(+72)	0.00(+62)
6	96.35(-98)	3.49(+158)	0.14(+54)	0.02(-47)	0.01(-108)
7	96.99(-23)	2.80(-46)	0.17(-59)	0.03(-65)	0.01(-61)
8	97.21(+58)	2.69(+97)	0.10(+129)	0.01(-178)	0.00(+20)
9	97.22(+129)	2.69(-113)	0.08(+20)	0.01(+123)	0.09(+3)
10	98.27(-152)	1.63(+3)	0.05(-112)	0.00(+119)	0.00(-53)
11	98.28(-10)	1.67(-6)	0.00(+89)	0.04(+145)	0.01(+171)
12	97.95(-26)	2.04(-45)	0.00(-316)	0.00(+12)	0.00(+138)
13	98.78(+2)	1.20(+17)	0.01(-36)	0.01(+122)	0.01(-130)
14	97.08(+68)	2.84(+122)	0.08(+142)	0.00(-150)	0.01(+63)
15	96.10(+146)	3.68(-76)	0.23(+66)	0.02(-133)	0.00(+87)
16	94.54(+130)	5.05(+92)	0.36(-49)	0.06(-158)	0.00(+86)
17	95.69(-62)	4.07(-124)	0.21(-169)	0.01(-169)	0.00(-176)
18	96.73(+20)	3.05(+37)	0.16(+48)	0.03(+62)	0.01(+72)
19	99.59(-106)	0.33(+78)	0.05(-81)	0.01(+172)	0.00(-32)
20	99.92(-160)	0.01(-144)	0.06(-81)	0.01(-152)	0.00(+180)
21	99.69(+176)	0.30(-50)	0.01(+92)	0.00(+155)	0.00(+56)
22	97.90(-157)	2.03(-13)	0.06(-176)	0.01(-106)	0.00(-166)
23	95.57(-102)	4.29(+154)	0.12(+39)	0.02(-92)	0.00(+174)
24	94.03(-29)	5.34(-85)	0.47(+120)	0.03(-147)	0.05(-146)
25	87.22(+43)	10.32(+72)	1.66(+106)	0.22(+144)	0.02(+157)
26	87.90(+120)	10.95(-134)	0.97(-1[illegible])	0.16(+107)	0.02(-115)
27	92.87(-154)	6.[illegible](+33)	0.51(-139)	0.[illegible](+57)	0.10(-43)
28	97.20(+10[illegible]	2.[illegible](-141)	0.0[illegible](+83)	0.[illegible](+146)	0.00(+110)
29	96.49(+[illegible]	[illegible](+113)	0.[illegible](+156)	[illegible](+76)	0.00(+100)
30	95.04(+[illegible]	4.[illegible](+27)	0.41(+[illegible]	[illegible]	0.02(+64)
31	94.41(+69)	[illegible](+119)	0.13(+151)	[illegible](+125)	0.02(+93)
32	92.29(+111)	7.[illegible](-155)	0.40(-[illegible]	0.0[illegible](+85)	0.00(-156)
33	83.77(+176)	[illegible](-36)	2.10(+129)	0.39(-66)	0.04(+128)
34	77.46(-117)	[illegible](+109)	4.01(-13)	[illegible](-139)	0.09(+97)
35	74.52(-30)	[illegible](-101)	4.[illegible](-151)	0.[illegible](+163)	0.12(+109)
36	79.41(+[illegible]	[illegible]	3.[illegible](+[illegible])	0.[illegible](+133)	0.07(+148)

* Ocean gage.

Table 11

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 1.00 sec, Ocean Wave Height = 0.130 ft

60-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 1.00 sec	First Harmonic 0.50 sec	Second Harmonic 0.33 sec	Third Harmonic 0.25 sec	Fourth Harmonic 0.20 sec
1*	98.82(+91)	0.97(+110)	0.17(+137)	0.04(-141)	0.00(+25)
2*	99.03(+122)	0.94(-111)	0.01(-31)	0.02(+88)	0.00(-99)
3	99.51(+139)	0.42(+112)	0.07(-177)	0.00(-177)	0.00(+97)
4	99.22(+151)	0.75(-87)	0.02(+34)	0.01(-155)	0.00(-108)
5	98.06(-144)	1.80(+57)	0.14(-115)	0.00(+110)	0.00(-12)
6	95.54(-60)	4.21(-114)	0.24(-159)	0.01(+157)	0.00(-173)
7	96.69(+19)	3.09(+35)	0.18(+50)	0.03(+81)	0.00(+149)
8	97.13(+93)	2.78(+174)	0.07(-105)	0.02(+33)	0.00(+88)
9	97.41(+164)	2.52(-33)	0.07(+134)	0.01(-69)	0.00(+104)
10	98.07(-108)	1.88(+140)	0.03(+15)	0.01(-89)	0.00(-129)
11	98.38(+15)	1.55(+54)	0.04(-143)	0.02(-41)	0.01(-45)
12	97.35(-28)	2.10(-53)	0.06(-15)	0.00(-65)	0.00(+52)
13	98.49(-20)	1.48(-27)	0.03(-101)	0.00(+131)	0.00(+167)
14	98.25(+27)	1.69(+49)	0.05(+78)	0.00(+49)	0.00(+138)
15	96.94(+104)	2.88(-156)	0.16(-58)	0.02(+70)	0.00(-76)
16	94.21(-169)	5.27(+9)	0.47(-169	0.04(+31)	0.00(-117)
17	94.33(-92)	5.36(+164)	0.27(+67)	0.03(-25)	0.01(-108)
18	95.81(-19)	3.97(-44)	0.19(68)	0.02(-73)	0.00(-81)
19	99.66(-140)	0.29(-73)	0.04(+85)	0.01(-16)	0.00(-149)
20	99.84(-180)	0.04(+90)	0.10(+9)	0.01(-98)	0.00(+10)
21	99.49(+178)	0.47(-29)	0.02(+25)	0.02(+143)	0.01(+16)
22	96.75(-135)	3.07(+70)	0.19(82)	0.00(+7)	0.00(-170)
23	93.83(-70)	5.91(-149)	0.24(+151)	0.03(+87)	0.00(-2)
24	89.48(+3)	9.50(-9)	0.84(-13)	0.15(-3)	0.03(+37)
25	88.38(+83)	10.37(+153)	1.12(-132)	0.11(-39)	0.02(+60)
26	91.54(+70)	7.73(-33)	0.65(+132)	0.08(-56)	0.01(+138)
27	93.51(-112)	6.03(+122)	0.40(-8)	0.05(-104)	0.01(+171)
28	97.23(+145)	2.74(-71)	0.01(-103)	0.01(+11)	0.00(-107)
29	97.09(+84)	2.77(+139)	0.11(+166)	0.03(+107)	0.01(+39)
30	96.63(+47)	3.24(+68)	0.06(+68)	0.03(+67)	0.03(+62)
31	93.73(+47)	5.76(+81)	0.45(+119)	0.04(+129)	0.03(+97)
32	93.23(+30)	6.32(+149)	0.42(-160)	0.02(-13)	0.01(+33)
33	86.99(+145)	11.76(-91)	1.03(+38)	0.21(+169)	0.01(+4)
34	77.45(-150)	18.10(+39)	3.71(-122)	0.66(+84)	0.08(-66)
35	74.32(-76)	19.69(-170)	4.41(+104)	0.90(+21)	0.18(-70)
36	75.96(+6)	18.54(-8)	4.34(-13)	0.98(-15)	0.18(-19)

* Ocean gage.

Distribut[illegible]

Incident [illegible]

Percent [illegible]

Gage	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1*	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2*	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	95.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	94.1[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13	93.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	93.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	91.1[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	87.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	90.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18	91.6[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	95.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23	89.0[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29	91.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
33	73.2[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	67.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

* Ocean ga[illegible]

Table 15

Distribution of Wave Energy Through at the Experimental Facility, 30-deg Incident Direction, Period [illegible] sec, Ocean Wave Height = 0.211 ft 60-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 1.00 sec	First Harmonic 0.50 sec	Second Harmonic 0.33 sec	Third Harmonic 0.25 sec	Fourth Harmonic 0.20 sec
1*	[illegible]	[illegible]	0.06(-163)	0.12(-9)	0.01(+92)
2*	[illegible]	1.68(-105)	0.05(-27)	0.05(+105)	0.00(-137)
3	98.36(+133)	1.57(+103)	0.07(+121)	0.00(+25)	0.00(+161)
4	[illegible]	3.57(-117)	0.33(+3)	0.07(+158)	0.01(-39)
5	94.99(-[illegible])	4.95(+58)	0.42(-92)	0.04(+131)	0.00(+2)
6	91.29(-21)	7.95(-142)	0.74(+162)	0.06(+125)	0.00(+52)
7	94.37(+6)	5.29(+6)	0.32(+16)	0.01(+17)	0.00(+4)
8	92.39(+33)	7.07(+146)	0.49(-136)	0.03(-7)	0.01(+11[illegible])
9	93.[illegible]	5.16(-50)	0.24(+109)	0.04(-66)	0.00(+114)
10	94.91(-115)	4.82(+115)	0.24(-8)	0.03(-106)	0.00(-154)
11	95.15(+13)	4.73(+57)	0.06(+89)	0.01(-104)	0.01(-2)
12	94.15(-29)	5.21(-58)	0.58(-76)	0.06(-116)	0.01(-129)
13	93.62(-15)	5.90(-27)	0.41(-36)	0.06(-52)	0.01(-50)
14	97.43(+18)	2.42(+51)	0.14(+37)	0.01(+38)	0.00(-159)
15	91.12(+39)	8.30(+168)	0.53(-105)	0.04(+22)	0.01(+83)
16	89.15(+174)	9.52(-32)	1.22(+135)	0.10(-64)	0.01(+36)
17	89.50(-103)	9.52(+135)	0.86(+19)	0.11(-81)	0.01(-170)
18	91.40(-30)	7.55(-63)	0.90(-92)	0.12(-105)	0.03(-104)
19	99.05(-141)	0.82(-64)	0.11(+41)	0.02(-110)	0.00(+161)
20	99.75(-178)	0.13(+36)	0.06(+46)	0.01(+151)	0.00(-32)
21	98.88(+174)	1.03(-45)	0.08(+128)	0.01(-103)	0.00(+[illegible])
22	92.55(-[illegible])	[illegible](+33)	0.65(-122)	0.03(+92)	0.01(-64)
23	85.20(-39)	13.35(+172)	1.72(+80)	0.21(-19)	0.02(-131)
24	81.41(-22)	16.[illegible](-57)	2.65(-88)	0.45(-104)	0.07(-127)
25	77.[illegible](+67)	18.36(+101)	3.58(+147)	0.56(-167)	0.08(-129)
26	[illegible]	[illegible](-[illegible])	2.52(+60)	0.32(-157)	0.03(-22)
27	[illegible]	[illegible](+84)	1.83(-51)	0.20(-177)	0.01(+48)
28	[illegible]	[illegible]	0.10(+38)	0.13(-79)	0.01(-104)
29	[illegible]	[illegible]	0.39(+178)	0.01(+166)	0.00(+59)
30	[illegible]	[illegible]	0.50(+124)	0.03(+69)	0.00(+78)
31	[illegible]	[illegible]	1.47(+96)	0.27(+111)	0.01(+[illegible])
32	[illegible]	[illegible]	[illegible](+171)	0.34(-130)	0.03(-21)
33	[illegible]	[illegible]	4.81(-51)	0.86(+61)	0.15(+166)
34	[illegible]	[illegible]	6.63(+115)	1.90(-90)	0.51(+59)
35	[illegible]	[illegible]	7.56(-39)	2.29(-177)	0.72(+43)
36	[illegible]	[illegible]	[illegible](-135)	2.29(+175)	0.65(+[illegible])

* Ocean gage

Table 5

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, [illegible] 1.50 sec, Ocean Wave Height = 0.070 ft

[illegible] Wastewater Mode with Shoreline

Percent [illegible] and Phase Angle (deg), Arrangement No. 1

Gage	Fundamental Period 1.5[illegible] sec	[illegible]	Second Harmonic 0.50 sec	Third Harmonic 0.35 sec	Fourth Harmonic 0.30 sec
1*	97.41(+8[illegible]	[illegible](-[illegible])	0.08(+20)	[illegible].05(+46)	0.03(+158)
2*	99.[illegible]	[illegible]	0.19(+33)	[illegible].01(-168)	0.00(+66)
3	98.[illegible]	[illegible](+)	[illegible].01(+2[illegible]	[illegible].02(-117)	0.00(+96)
4	[illegible].1[illegible]	[illegible]	[illegible]	[illegible].[illegible](+9[illegible]	0.01(+78)
5	[illegible]	[illegible]	[illegible].[illegible](-1[illegible]	[illegible].[illegible](-175)	0.00(+178)
6	97.[illegible]	[illegible](-[illegible])	0.0[illegible](-[illegible])	[illegible].[illegible](-121)	0.01(+111)
7	99.[illegible]	[illegible](+[illegible])	0.0[illegible](+[illegible])	0.01(+123)	0.01(+23)
8	99.[illegible]	[illegible](-[illegible])	0.12(-[illegible]	[illegible].00(+58)	0.01(-24)
9	[illegible].[illegible](+[illegible]	[illegible]	0.31(-172[illegible]	[illegible].[illegible](+22)	0.01(+51)
10	99.[illegible]	[illegible](-[illegible])	0.04(+[illegible]	0.10(-84)	0.01(-103)
11	[illegible].[illegible]	[illegible]	0.00(-17[illegible]	0.04(+123)	0.01(+52)
12	[illegible].[illegible]	[illegible]	0.24(+146)	0.07(-81)	0.08(+111)
13	[illegible].[illegible]	[illegible]	0.01(-89)	0.04(-7)	0.02(-91)
14	[illegible].[illegible]	[illegible]	0.04(-65)	0.04(-3)	0.00(-104)
15	[illegible].[illegible]	[illegible]	0.03(-152)	0.01(-40)	0.00(-74)
16	[illegible].[illegible]	[illegible]	0.09(+[illegible]	0.04(-111)	0.01(-159)
17	[illegible].[illegible]	[illegible]	[illegible].[illegible](-[illegible]	[illegible].01(-131)	0.00(+164)
18	[illegible].[illegible]	[illegible]	[illegible].[illegible](-[illegible]	[illegible].[illegible](-[illegible]	0.00(-106)
19	[illegible].[illegible]	[illegible]	0.[illegible](-130)	0.00(-39)	0.01(+30)
20	99.[illegible]	[illegible]	0.0[illegible](+140)	0.09(+141)	0.01(+107)
21	[illegible].[illegible]	[illegible]	0.07(+[illegible]	[illegible].04(-91)	0.02(-16)
22	98.[illegible]	[illegible]	[illegible].[illegible](-[illegible]	0.[illegible](+119)	0.06(-99)
23	[illegible].[illegible]	[illegible]	[illegible].04(+[illegible]	0.13(+107)	0.06(+168)
24	[illegible].[illegible]	[illegible]	0.13(-[illegible]	[illegible].[illegible](+92)	0.00(-99)
25	[illegible].[illegible]	[illegible]	0.2[illegible]	[illegible].[illegible](-89)	0.00(+177)
26	[illegible].[illegible]	[illegible]	[illegible].50(-[illegible]	[illegible].[illegible](+156)	0.01(+91)
27	89.[illegible]	[illegible]	1.[illegible]	[illegible].[illegible](+[illegible]	0.01(+24)
28	[illegible].[illegible]	[illegible]	[illegible].[illegible](-[illegible]	[illegible].01(+55)	0.02(-2)
29	98.[illegible]	[illegible]	[illegible].[illegible](-[illegible]	[illegible].[illegible](-[illegible]	0.01(-2)
30	[illegible].[illegible]	[illegible]	[illegible].[illegible](-[illegible]	[illegible].01(-[illegible]	0.01(-114)
31	[illegible].[illegible]	[illegible]	[illegible].[illegible](+[illegible]	[illegible].[illegible](-[illegible]	0.02(-31)
32	[illegible].[illegible]	[illegible]	[illegible].[illegible](-[illegible]	[illegible].[illegible](+[illegible]	0.02(+119)
33	[illegible].[illegible]	[illegible]	[illegible].[illegible](-[illegible]	[illegible].[illegible](-[illegible]	0.01(+173)
34	[illegible].[illegible]	[illegible]	1.[illegible](+[illegible]	[illegible].[illegible](+[illegible]	0.06(+135)
35	[illegible].[illegible]	[illegible]	4.[illegible](-[illegible]	[illegible].[illegible](-[illegible]	0.16(+86)
36	[illegible].[illegible]	[illegible]	3.[illegible](+[illegible]	[illegible].[illegible](+[illegible]	0.12(-50)

* Ocean gages

Table 15

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 1.50 sec, Ocean Wave Height = 0.070 ft

66-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 1.50 sec	First Harmonic 0.75 sec	Second Harmonic 0.50 sec	Third Harmonic 0.38 sec	Fourth Harmonic 0.30 sec
1*	97.32(+81)	2.77(-87)	0.00(+146)	0.00(-75)	0.01(-173)
2*	99.17(+97)	0.66(-9)	0.17(+131)	0.00(-79)	0.00(+98)
3	95.88(-94)	3.95(+177)	0.11(+91)	0.01(-174)	0.06(+55)
4	94.18(-89)	5.73(-174)	0.06(+99)	0.01(+77)	0.03(+118)
5	98.05(-55)	1.90(-66)	0.02(+16)	0.02(+57)	0.00(+77)
6	99.47(-16)	0.46(+16)	0.07(-11)	0.00(-43)	0.00(+146)
7	99.84(+34)	0.07(+127)	0.09(-89)	0.01(-38)	0.00(+50)
8	99.77(+86)	0.15(+9)	0.08(-169)	0.00(+82)	0.00(-2)
9	99.95(+137)	1.03(+171)	0.01(-157)	0.02(-127)	0.00(-37)
10	97.60(+175)	2.17(-89)	0.23(+26)	0.00(+70)	0.00(-77)
11	96.62(+56)	3.29(+129)	0.02(-139)	0.05(+134)	0.01(-99)
12	96.38(+27)	3.53(+67)	0.01(-115)	0.03(-36)	0.06(-21)
13	98.04(+33)	1.81(+97)	0.02(+143)	0.02(+168)	0.12(+42)
14	98.67(+52)	1.18(+152)	0.13(-148)	0.01(+80)	0.01(+41)
15	99.52(+94)	0.27(+149)	0.20(+119)	0.00(+145)	0.00(-97)
16	98.19(+144)	1.70(-126)	0.11(-17)	0.01(+170)	0.00(-124)
17	96.83(-163)	3.11(-34)	0.05(+71)	0.01(+177)	0.00(+27)
18	94.38(-116)	5.07(+61)	0.54(-103)	0.04(+92)	0.00(+23)
19	97.45(-177)	2.50(+35)	0.02(-111)	0.02(+17)	0.01(+64)
20	96.62(+163)	3.29(-2)	0.06(+120)	0.02(-163)	0.02(-8)
21	97.41(+161)	2.32(-4)	0.05(-122)	0.11(-69)	0.12(-29)
22	99.74(-177)	0.21(+6)	0.02(-35)	0.00(+115)	0.02(-56)
23	99.65(-145)	0.98(+2)	0.35(-164)	0.02(+103)	0.00(+10)
24	94.24(-104)	5.71(+93)	0.03(-19)	0.02(+113)	0.01(-14)
25	92.18(-47)	6.68(-156)	1.08(+122)	0.07(+37)	0.02(-21)
26	89.57(+3)	9.71(-42)	0.67(-45)	0.05(-71)	0.01(-152)
27	88.49(+49)	10.99(+55)	0.50(+72)	0.03(+77)	0.00(+85)
28	99.20(+4)	0.67(+106)	0.06(+158)	0.02(-100)	0.05(-147)
29	98.92(-41)	1.06(-9)	0.01(+60)	0.01(+168)	0.00(+94)
30	99.33(-5.)	0.56(-102)	0.08(-91)	0.01(+116)	0.03(+5)
31	99.26(-45)	0.71(-34)	0.14(+103)	0.06(-10)	0.03(-60)
32	98.08(-33)	1.80(-101)	0.11(+152)	0.10(+98)	0.02(+93)
33	94.69(-2)	4.73(-64)	0.45(-89)	0.03(-128)	0.00(+125)
34	86.81(+37)	11.32(+23)	1.75(+14)	0.22(+23)	0.02(+5)
35	78.76(+92)	16.33(-131)	3.97(-162)	0.61(-99)	0.14(-30)
36	75.21(+139)	19.23(-126)	4.57(-21)	0.75(+93)	0.23(-157)

* Ocean gage.

Table 5

Distribution of Wave Energy Throughout the Experimental Gage Array for

Incident Wave Crest, Period = 1.50 sec, Ocean Wave Height = [illegible] ft

45-deg Breakwater Angle with Shore Line

Percent of Total Energy and Phase Angle (deg), Arrangement No. [illegible]

Gage	Superharmonic Energy 1.0 sec	First Harmonic 1.50 sec	Second Harmonic 0.50 sec	Third Harmonic 0.38 sec	Fourth Harmonic 0.30 sec
1*	98.[illegible] (+4)	1.[illegible] (-106)	0.14 (-20)	0.05 (+145)	0.04 (+96)
2*	97.[illegible] (+[illegible])	1.[illegible] (-[illegible])	0.[illegible] (+[illegible])	0.02 (-36)	0.01 (+2[illegible])
3	98.[illegible] (-[illegible])	1.[illegible] (+124)	0.16 (-119)	0.01 (+133)	0.01 (-1[illegible])
4	9[illegible] (-[illegible])	[illegible] (+142)	0.14 (+[illegible])	0.03 (+50)	0.03 (+23)
5	88.[illegible] (-[illegible])	10.[illegible] (-161)	0.64 (+127)	0.04 (+54)	0.01 (-110)
6	9[illegible].[illegible] (-[illegible])	7.34 (-61)	0.37 (-[illegible])	0.02 (-134)	0.00 (-[illegible])
7	95.[illegible] (-[illegible])	4.[illegible] (+[illegible])	0.17 (+32)	0.00 (-36)	0.00 (-141)
8	98.[illegible] (+[illegible])	1.[illegible] (+154)	0.2[illegible] (-148)	0.02 (-33)	0.00 (+[illegible])
9	99.[illegible] (+[illegible])	[illegible] (-[illegible])	0.10 (+92)	0.01 (-131)	0.00 (+80)
10	99.[illegible] (+[illegible])	[illegible] (-122)	0.[illegible] (-[illegible])	0.[illegible] (+152)	0.00 (-[illegible])
11	89.[illegible] (+[illegible])	[illegible] (+63)	0.23 (+101)	0.03 (+12)	0.00 (-[illegible])
12	89.30 (+19)	9.[illegible] (+52)	0.41 (+34)	0.01 (+98)	0.00 (-11[illegible])
13	90.[illegible] (+32)	8.[illegible] (+[illegible])	0.15 (+159)	0.02 (+152)	0.[illegible] (+18)
14	99.[illegible] (+60)	3.[illegible] (+169)	0.36 (-161)	0.04 (-127)	0.00 (-[illegible])
15	93.97 (+155)	[illegible] (-114)	0.29 (+[illegible])	0.[illegible] (-109)	0.[illegible] (-[illegible])
16	99.53 (+163)	[illegible] (-50)	0.14 (-9)	0.04 (+103)	[illegible] (-1[illegible])
17	93.[illegible] (-[illegible])	1.[illegible] (-12)	0.0[illegible] (+152)	0.01 (-26)	0.[illegible] (-[illegible])
18	95.[illegible] (-[illegible])	[illegible] (+[illegible])	0.44 (-[illegible])	0.[illegible] (+142)	[illegible] (+[illegible])
19	9[illegible].[illegible] (-[illegible])	[illegible] (+[illegible])	0.01 (+[illegible])	0.01 (-[illegible])	[illegible] (+[illegible])
20	96.[illegible] (+[illegible])	[illegible] (-[illegible])	0.09 (+[illegible])	0.0[illegible] (+[illegible])	[illegible] (-[illegible])
21	94.61 (+[illegible])	5.34 (-44)	0.03 (+[illegible])	0.00 (-127)	0.02 (-[illegible])
22	95.[illegible] (+[illegible])	4.01 (-31)	0.[illegible] (+[illegible])	0.01 (-103)	0.00 (+[illegible])
23	97.[illegible] (-[illegible])	2.0[illegible] (+[illegible])	0.14 (-98)	0.01 (+45)	0.00 (-[illegible])
24	98.[illegible] (-[illegible])	0.9[illegible] (+44)	0.[illegible] (+[illegible])	0.03 (+54)	0.00 (+[illegible])
25	93.[illegible] (-[illegible])	5.[illegible] (+[illegible])	0.[illegible] (+[illegible])	0.0[illegible] (-[illegible])	0.[illegible] (+[illegible])
26	8[illegible].[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (+[illegible])	[illegible] (+[illegible])
27	[illegible].[illegible] (+[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])
28	[illegible].[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (+[illegible])	[illegible] (+[illegible])
29	[illegible].[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])
30	[illegible].[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])
31	[illegible].[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])
32	[illegible].[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (+[illegible])	[illegible] (+[illegible])	[illegible] (+[illegible])
33	[illegible].[illegible] (+[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])
34	[illegible].[illegible] (+[illegible])	[illegible] (+[illegible])	[illegible] (+[illegible])	[illegible] (+[illegible])	[illegible] (+[illegible])
35	[illegible].[illegible] (+[illegible])	[illegible] (+[illegible])	[illegible] (-[illegible])	[illegible] (+[illegible])	[illegible] (+[illegible])
36	[illegible].[illegible] (+[illegible])	[illegible] (-[illegible])	[illegible] (-[illegible])	[illegible] (+[illegible])	[illegible] (-[illegible])

[illegible] Page

Table 1

Distribution of Wave Energy Throughout the Experimental Facility, 30-deg Incident Direction, Period = 1.50 sec, Ocean Wave Height = 0.106 ft

30-deg Breakwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 1.50 sec	First Harmonic 0.75 sec	Second Harmonic 0.50 sec	Third Harmonic 0.38 sec	Fourth Harmonic 0.30 sec
1*	94.36(+89)	5.01(-76)	0.08(+11)	0.01(-86)	0.03(+90)
2*	97.13(+103)	1.94(-19)	0.24(+110)	0.03(-4)	0.01(+48)
3	91.03(-88)	8.86(+179)	0.06(-175)	0.03(-53)	0.03(+7)
4	89.31(-87)	9.57(-162)	0.83(+88)	0.02(+26)	0.00(+158)
5	93.09(-54)	6.78(-68)	0.15(-75)	0.00(+102)	0.00(+74)
6	97.43(-13)	2.41(+30)	0.14(+8)	0.01(-52)	0.00(-115)
7	98.67(+39)	1.04(+132)	0.26(-133)	0.03(-115)	0.00(+111)
8	99.71(+91)	0.17(-85)	0.10(+81)	0.01(-165)	0.00(-19)
9	98.99(+149)	0.85(+149)	0.15(-75)	0.01(+97)	0.00(-14)
10	95.47(+179)	3.98(-39)	0.19(+12)	0.03(+133)	0.00(-27)
11	93.72(+56)	6.05(+138)	0.12(-130)	0.08(-11)	0.02(+104)
12	89.84(+34)	9.13(+76)	0.85(+112)	0.16(+125)	0.01(-74)
13	90.68(+35)	8.72(+93)	0.57(+162)	0.01(-40)	0.02(+12)
14	96.35(+56)	3.51(+164)	0.13(-160)	0.01(-135)	0.00(-102)
15	99.18(+95)	0.57(-139)	0.42(+82)	0.02(+133)	0.01(-23)
16	98.38(+147)	1.38(-113)	0.24(-40)	0.04(+87)	0.00(+20)
17	97.54(-159)	2.33(-23)	0.12(+138)	0.02(-56)	0.00(+73)
18	93.05(-112)	6.51(+64)	0.35(-106)	0.07(+98)	0.01(-56)
19	94.73(-175)	5.23(+39)	0.01(-102)	0.00(-152)	0.03(+70)
20	94.35(+162)	5.00(+1)	0.05(+23)	0.01(93)	0.06(-151)
21	94.97(+163)	4.99(+4)	0.06(+107)	0.04(-128)	0.03(-58)
22	99.40(-176)	0.34(+35)	0.13(-75)	0.04(+125)	0.01(+3)
23	98.42(-142)	1.13(+11)	0.38(+177)	0.07(+31)	0.00(-161)
24	93.46(-99)	5.38(+9))	0.54(-46)	0.02(+178)	0.00(+39)
25	87.58(-46)	10.99(-159)	1.10(+112)	0.19(+38)	0.04(-38)
26	80.38(+61)	13.49(-47)	5.33(-68)	0.63(-81)	0.11(-105)
27	78.30(+51)	13.23(+49)	8.13(+83)	0.45(+108)	0.10(+144)
28	98.06(+72)	1.87(-43)	0.06(+146)	0.00(+152)	0.01(+103)
29	97.53(-43)	2.43(-47)	0.03(-46)	0.00(-67)	0.00(-19)
30	94.[illegible]	[illegible]	0.51(-114)	0.00(-57)	0.09(+99)
31	[illegible]	[illegible]	1.29(-113)	0.01(+121)	0.02(-13)
32	[illegible]	[illegible]	0.12(+144)	0.13(+40)	0.03(-7)
33	90.[illegible]	[illegible]	1.93(-108)	0.51(-152)	0.11(+164)
34	[illegible]	[illegible]	[illegible]	1.60(+18)	0.42(+30)
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 38

Distribution [illegible] throughout [illegible] Grading

Incident [illegible]

[illegible] Location Angle [illegible]

Percent [illegible] [illegible]

Cage	Percent [illegible] 1.5σ SE	[illegible]	Percent 0.01 SE	[illegible]	[illegible]
1*	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2*	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Table 19

Distribution of Wave Energy Throughout the Experimental Facility, T = 50,

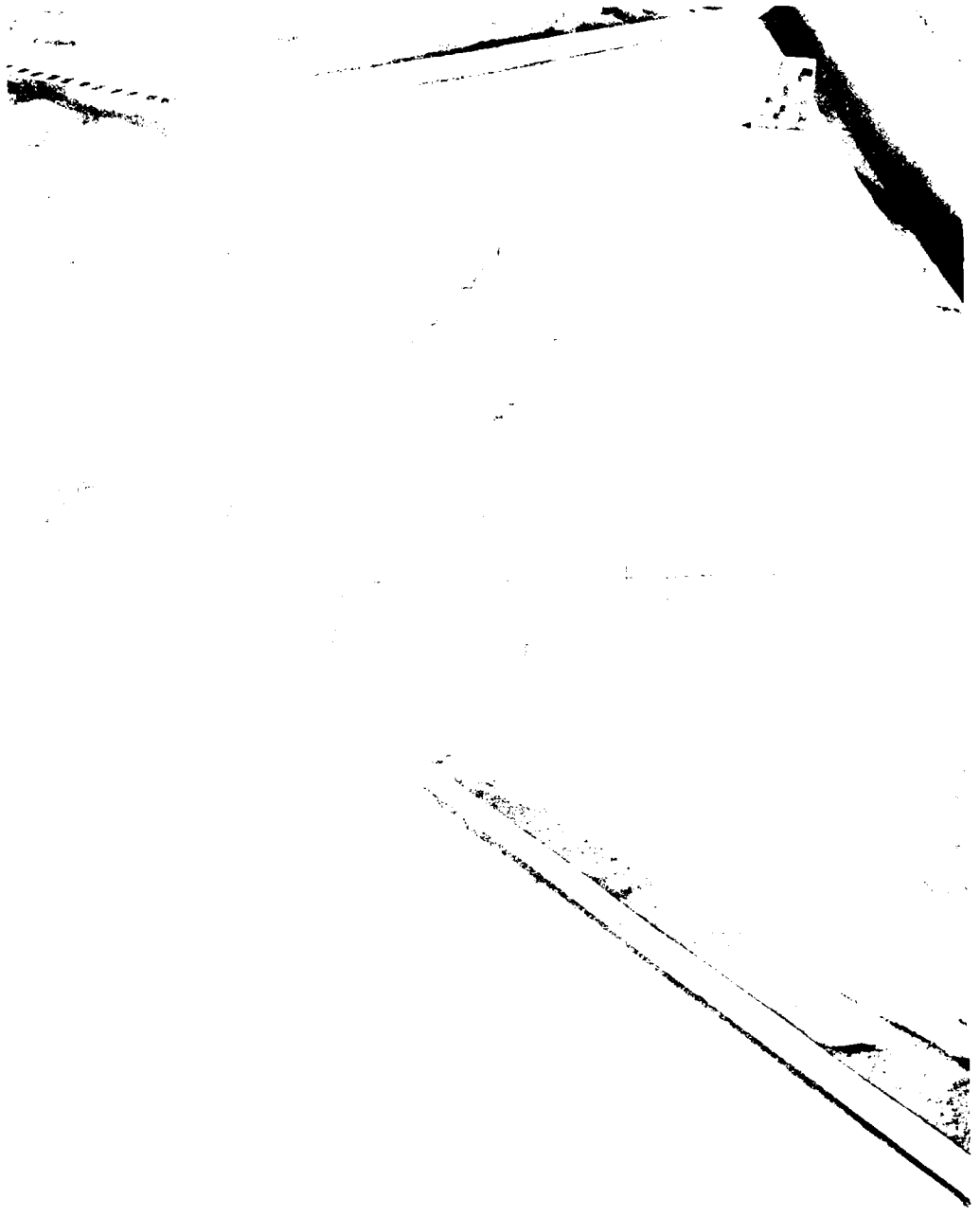
Incident Direction, Period = 1.50 sec, Ocean Wave Height = 0.547 ft

Gages on Underwater Angle with Shoreline

Percent of Total Energy and Phase Angle (deg), Arrangement No. 2

Gage	Fundamental Period 1.50 sec	First Harmonic 0.75 sec	Second Harmonic 0.50 sec	Third Harmonic 0.38 sec	Fourth Harmonic 0.30 sec
1*	93.12(+101)	6.37(-49)	0.48(+41)	0.02(-54)	0.01(+2)
2*	94.63(+113)	4.16(-2)	1.13(+130)	0.02(-51)	0.02(+55)
3	84.68(-73)	14.82(-169)	0.17(+171)	0.29(+172)	0.03(+178)
4	79.57(-73)	19.13(-144)	1.16(+100)	0.12(+59)	0.01(-65)
5	86.51(-45)	11.91(-57)	1.42(-37)	0.11(-112)	0.05(-127)
6	92.23(-6)	7.21(+40)	0.47(+73)	0.04(+107)	0.00(+114)
7	95.50(+47)	3.63(+142)	0.74(-142)	0.11(-95)	0.00(-58)
8	98.84(+101)	0.82(-97)	0.30(+77)	0.04(-158)	0.00(-51)
9	98.90(+150)	0.45(+176)	0.58(-69)	0.07(+73)	0.00(-1[illegible])
10	93.30(-170)	6.23(-70)	0.43(+101)	0.02(+178)	0.01(+32)
11	87.96(+68)	10.89(+156)	0.90(-94)	0.17(-55)	0.09(-45)
12	84.00(+44)	13.63(+93)	2.28(+128)	0.05(+176)	0.03(-169)
13	83.40(+45)	14.31(+102)	1.53(+163)	0.23(-120)	0.03(-37)
14	90.10(+60)	9.61(+171)	0.25(-115)	0.03(+12)	0.00(-171)
15	97.69(+102)	1.66(-102)	0.57(+37)	0.06(+140)	0.01(-116)
16	98.27(+155)	1.39(-73)	0.21(-93)	0.11(+57)	0.01(-162)
17	96.88(-152)	2.53(+10)	0.44(+149)	0.09(-21)	0.01(+177)
18	91.17(-102)	7.30(+82)	1.29(-59)	0.22(+165)	0.02(+12)
19	89.36(-165)	10.45(+60)	0.04(+2)	0.04(-84)	0.11(-38)
20	87.32(+169)	12.30(+16)	0.09(-102)	0.29(+48)	0.00(-50)
21	91.72(+171)	8.02(+18)	0.24(+168)	0.01(+89)	0.00(-148)
22	97.91(-171)	1.37(+33)	0.53(-109)	0.19(+66)	0.01(-135)
23	97.97(-136)	1.18(+45)	0.61(+133)	0.19(-29)	0.04(-168)
24	92.36(-92)	5.13(+107)	1.98(-31)	0.42(-139)	0.13(+135)
25	83.74(-36)	11.68(-147)	3.82(+140)	0.64(+75)	0.13(+11)
26	73.74(+13)	17.19(-36)	5.26(-56)	1.91(-69)	0.40(-83)
27	67.50(+66)	21.21(+63)	8.40(+94)	1.84(+129)	0.55(+168)
28	97.91(+15)	1.38(+79)	0.65(+151)	0.08(+155)	0.07(+104)
29	95.61(-19)	3.85(-10)	1.39(-22)	0.02(+106)	0.13(-138)
30	94.34(-[illegible])	3.93(-67)	0.92(-124)	0.32(-139)	0.09(-1[illegible])
31	94.03(-[illegible])	4.[illegible](-[illegible])	1.41(-109)	0.29(-157)	0.22(-137)
32	95.00(-[illegible])	3.[illegible](-[illegible])	1.03(+134)	0.01(+54)	0.[illegible](+4)
33	87.72(+[illegible])	9.[illegible](-[illegible])	2.11(-106)	1.34(-129)	0.43(-1[illegible])
34	79.49(+[illegible])	16.95(+23)	3.01(+26)	2.[illegible](+[illegible])	0.56(+[illegible])
35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Ocean [illegible]



[illegible]

[illegible] of breakwater at 60 deg to shoreline [illegible] the wave generator approaching from an [illegible] of 30 deg

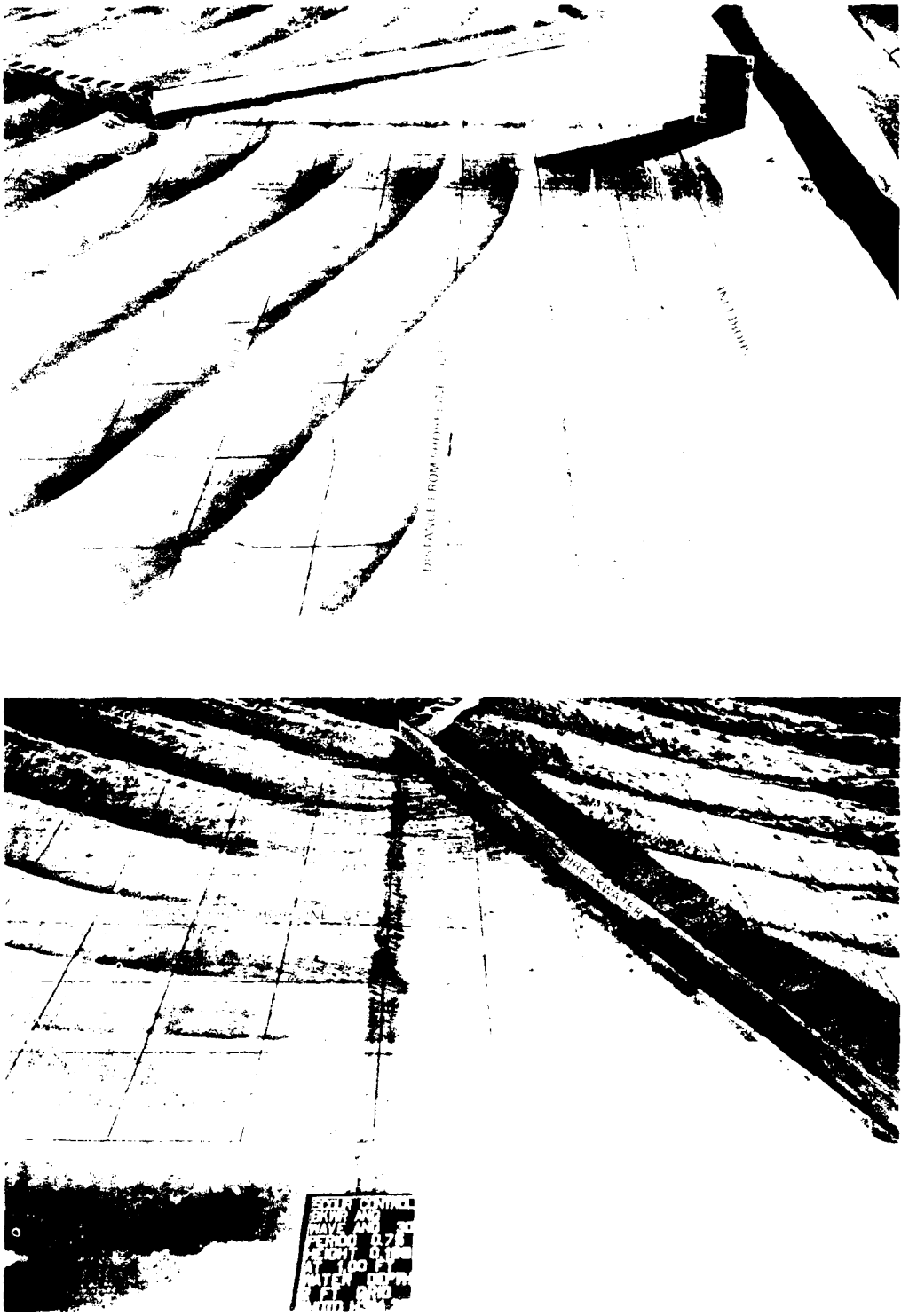


Photo 2. Typical wave pattern downwave of breakwater at 60 deg to shoreline for 0.75-sec, 0.168-ft high wave near the wave generator approaching from an incident direction of 30 deg

Photo
for

at 60 deg to shoreline
approaching from an

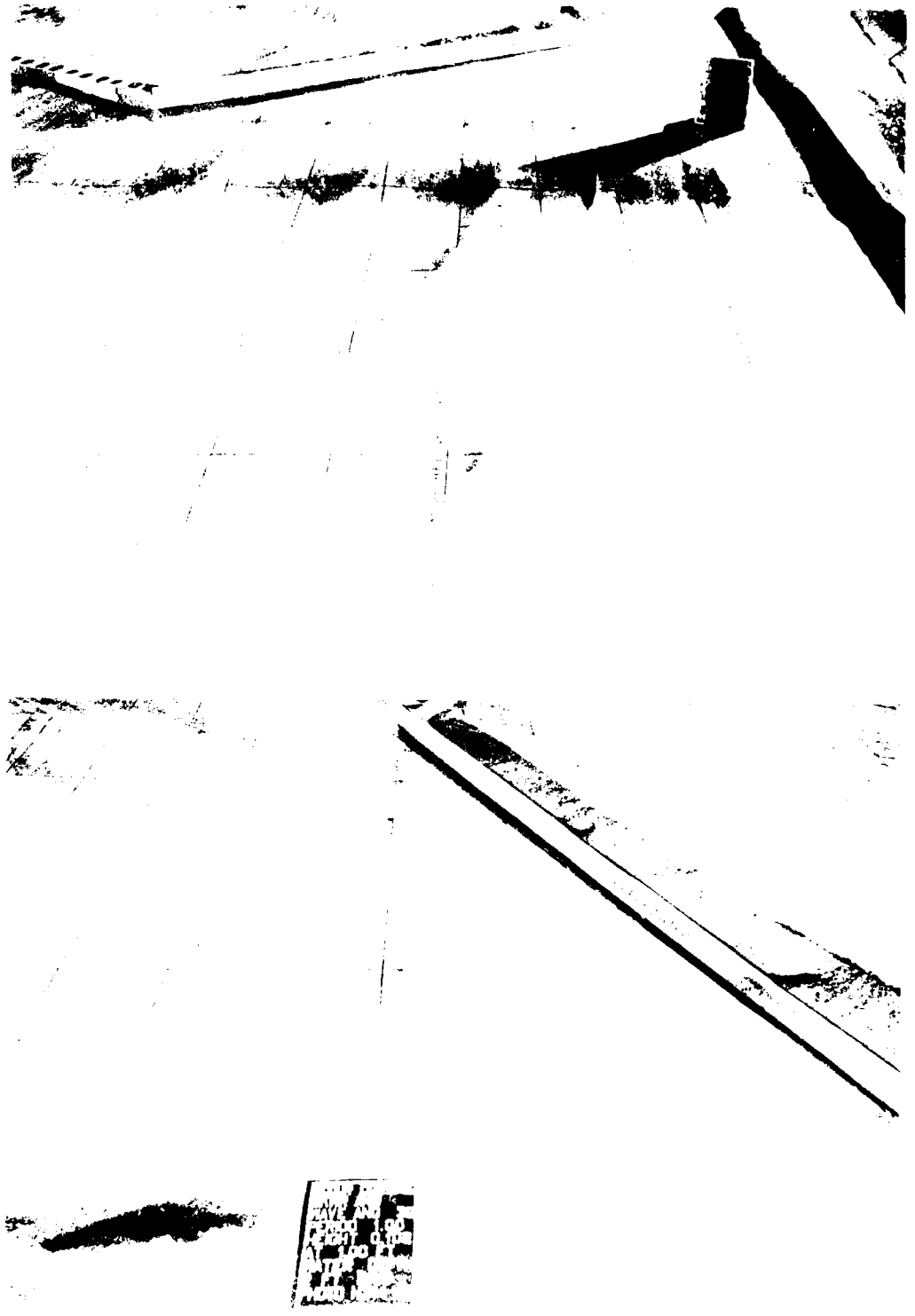


Photo 4. [illegible] pattern downwave of breakwater at 60 deg to shoreline for [illegible] near the wave generator approaching from an incident direction of 30 deg

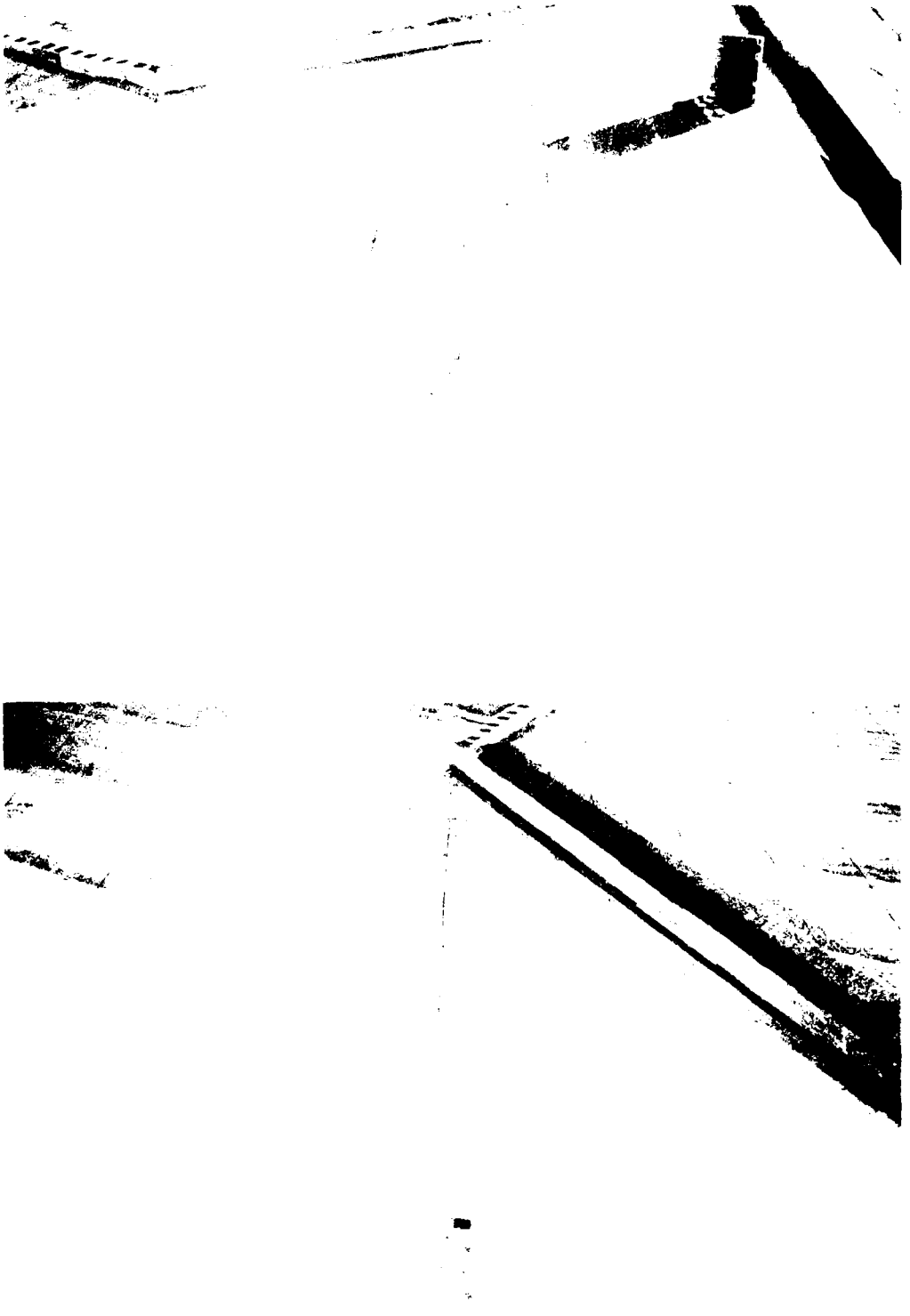


Photo 5. [illegible] downwave of breakwater at 60 deg to shoreline for 1.0-sec, [illegible] waves; the wave generator approaching from an [illegible] direction of 30 deg

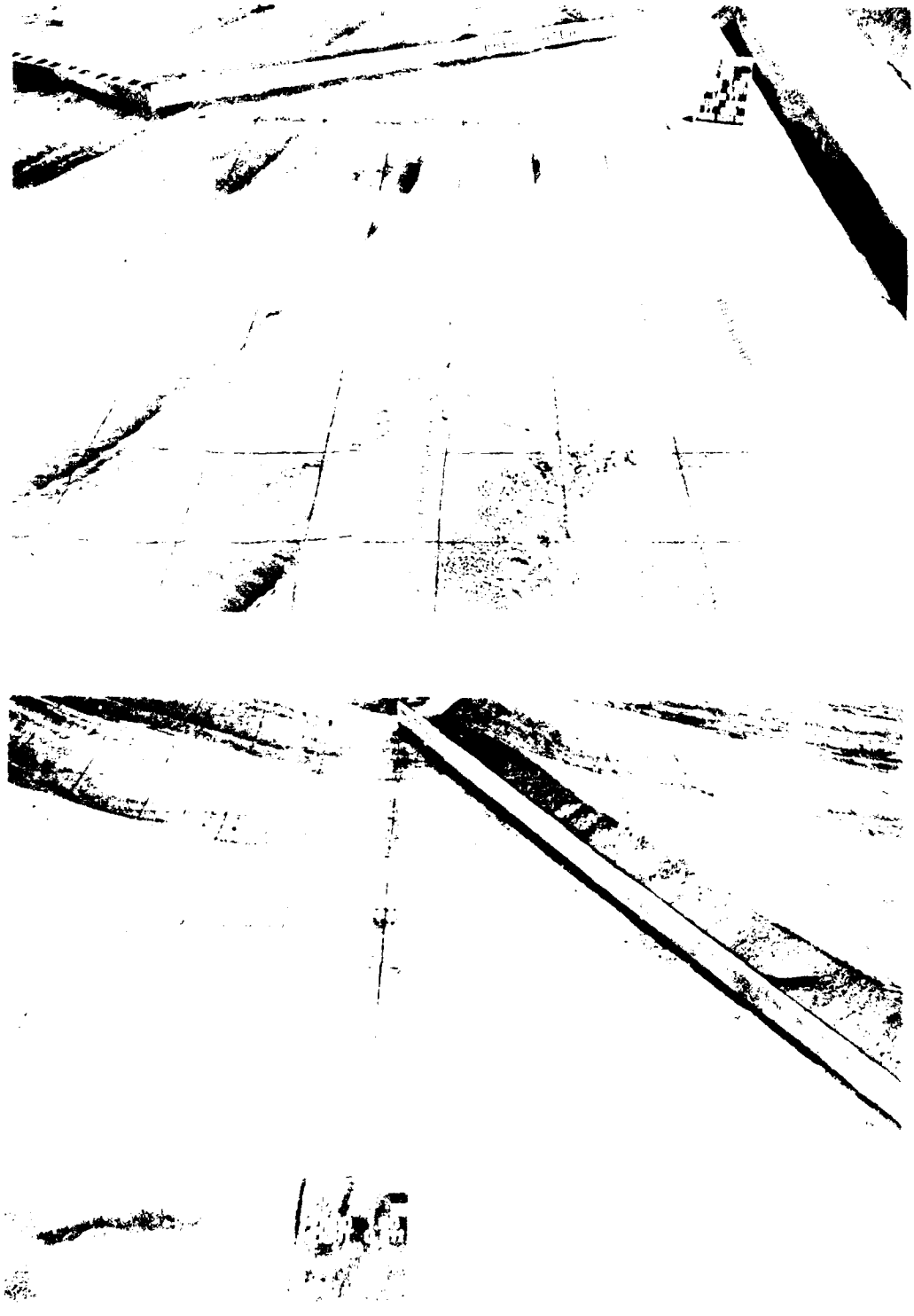


Photo 6. [illegible] f breakwater at 60 deg to shoreline for 1.0 [illegible] by wave generator approaching from an [illegible] direction of 30 deg

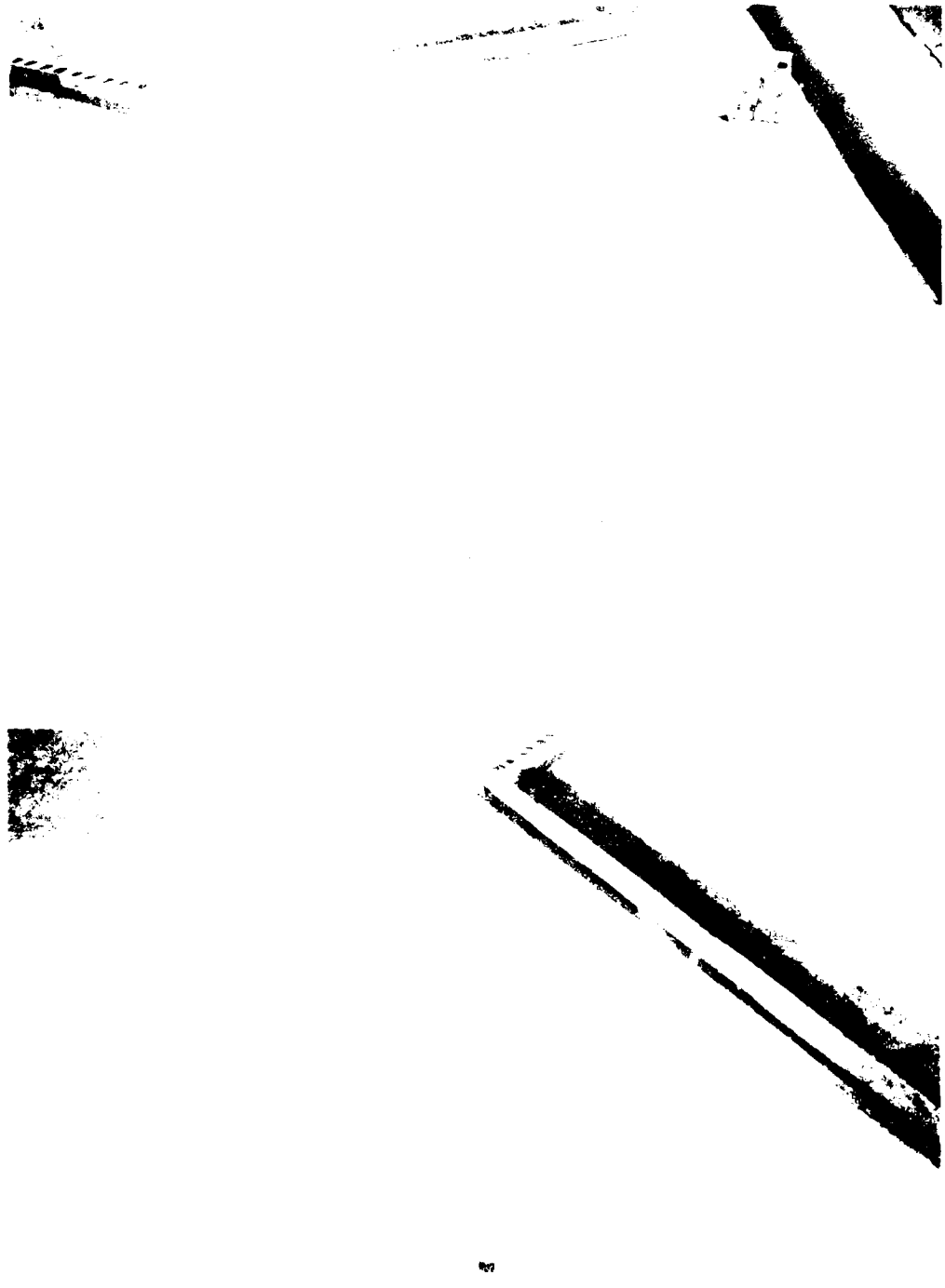


Photo [illegible]
for [illegible]

[illegible] at [illegible] to shoreline
[illegible] penetrator approaching from an
[illegible]

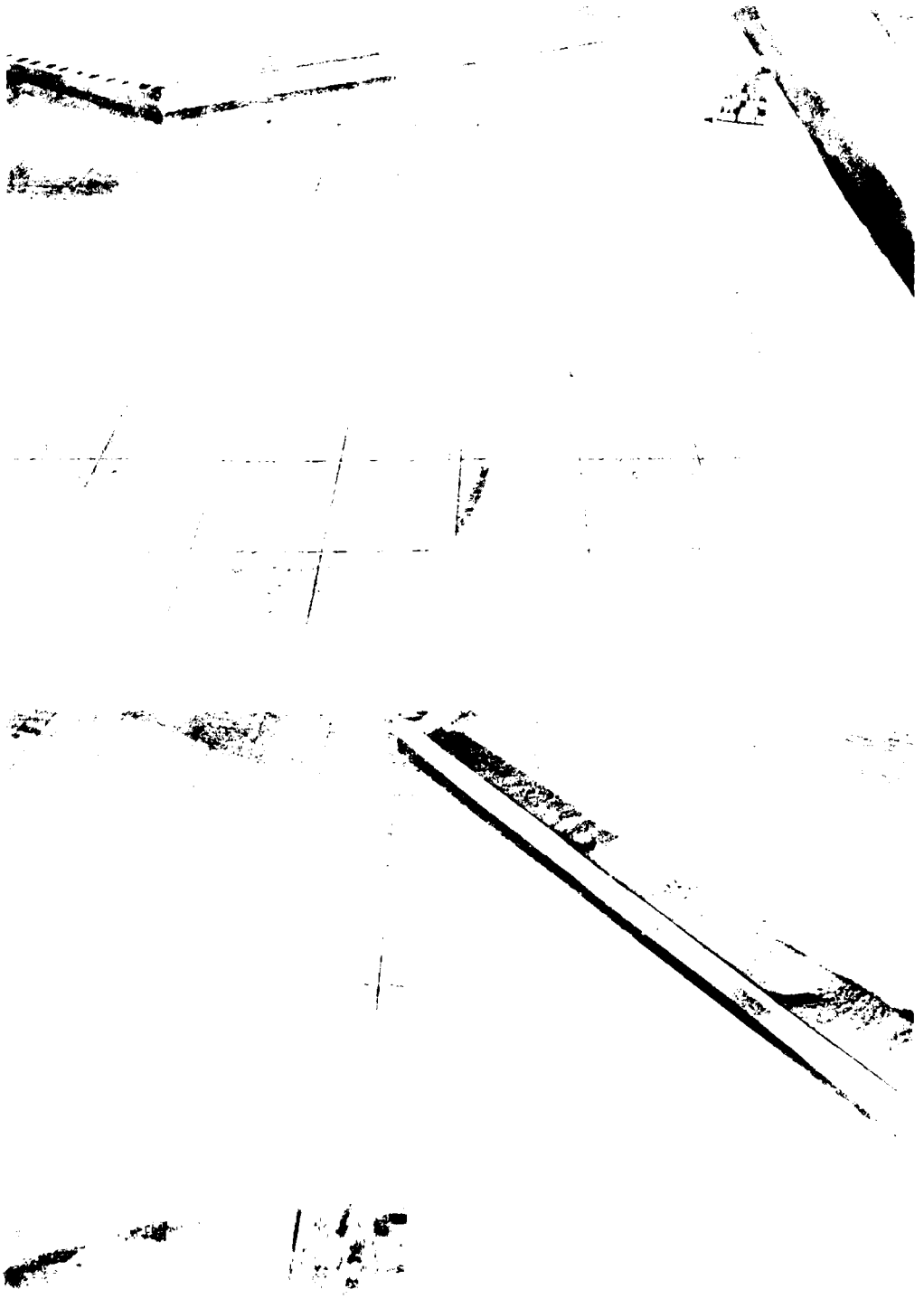


Photo
for 1

breakwater at 60 deg to shoreline
wave generator approaching from an
of 30 deg

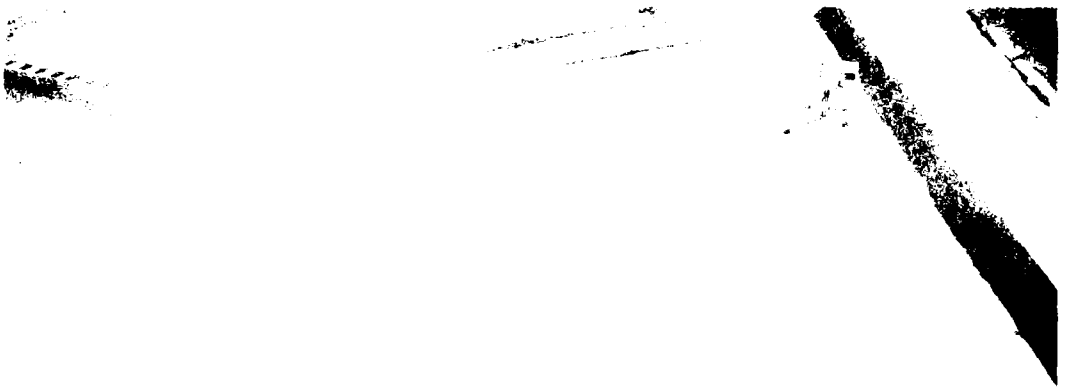


Photo
Ice

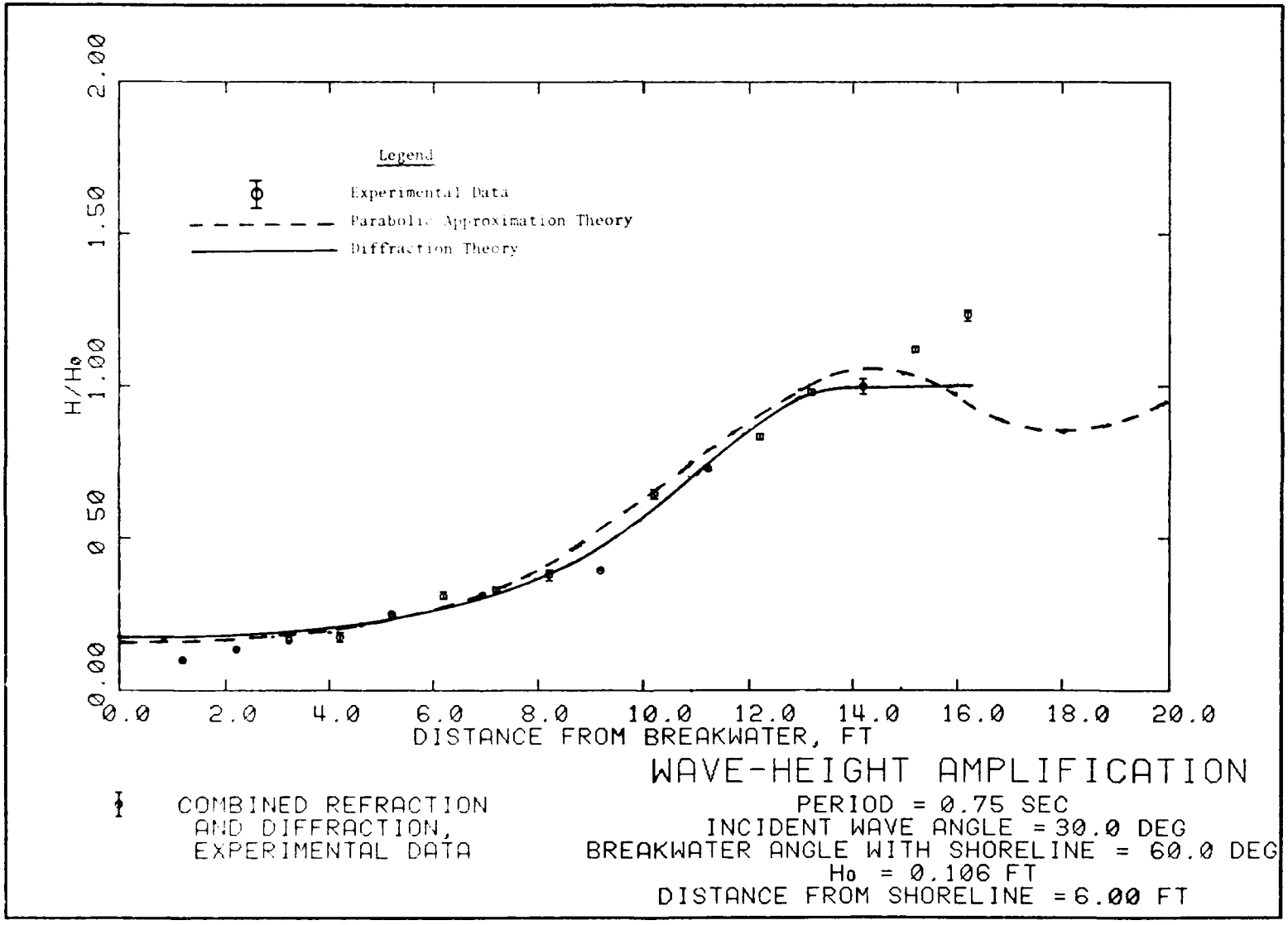
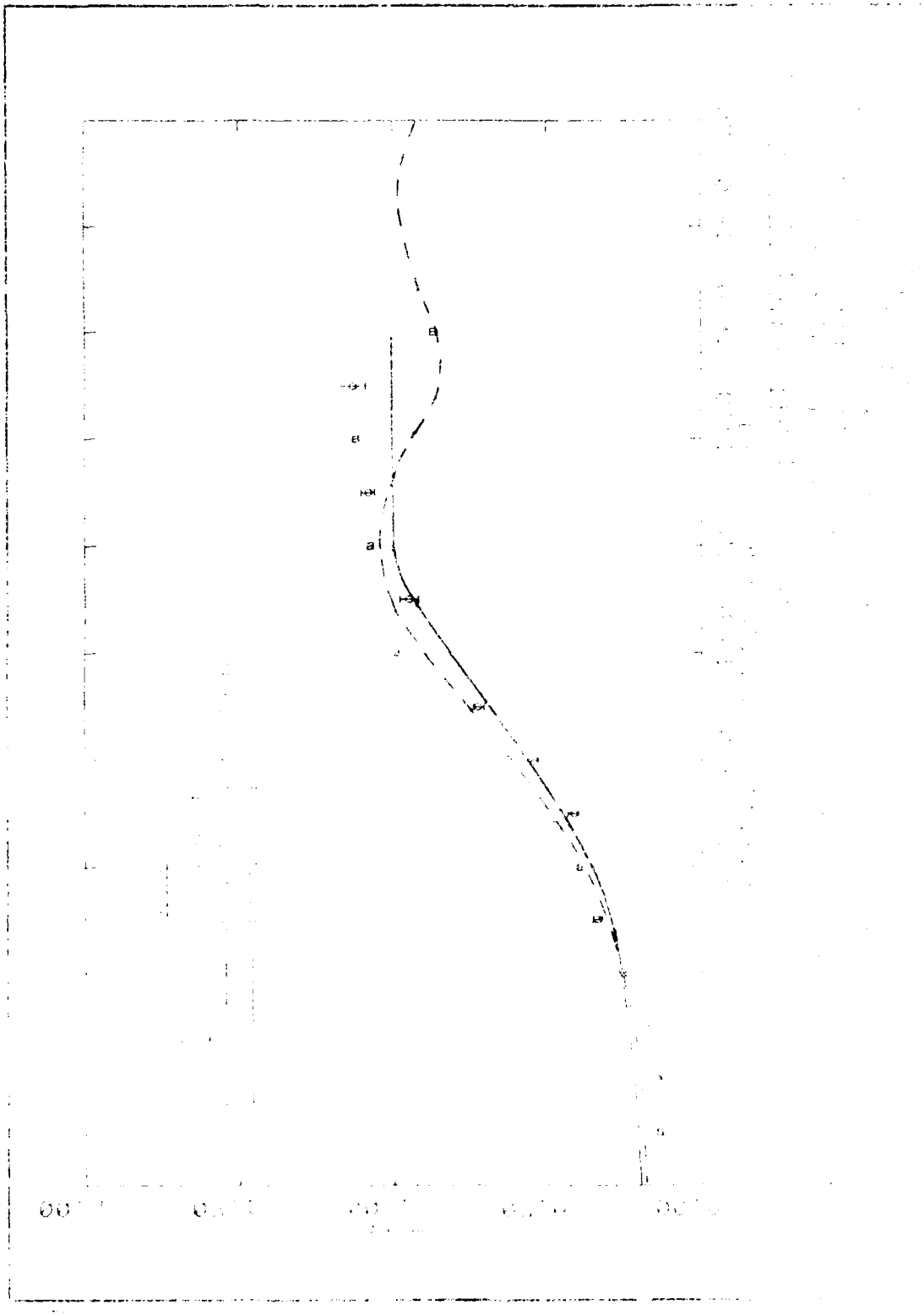
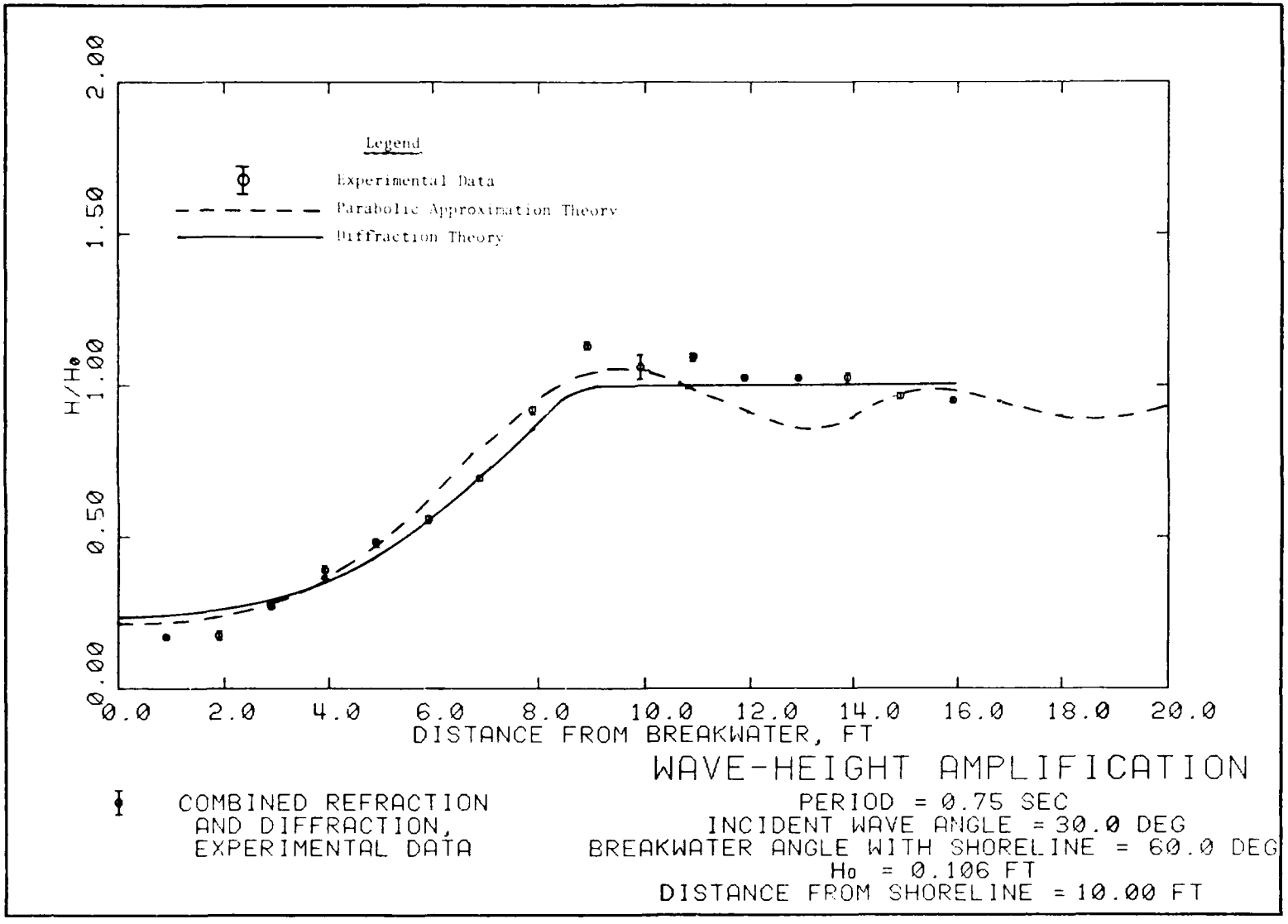

Legend
Experimental Data
Parabolic Approximation Theory
Diffraction Theory
H/Ho
0.00
0 50
1.00
1.50
2.00
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT
WAVE-HEIGHT AMPLIFICATION
COMBINED REFRACTION
AND DIFFRACTION,
EXPERIMENTAL DATA
PERIOD = 0.75 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.106 FT
DISTANCE FROM SHORELINE = 6.00 FT

PLATE 1



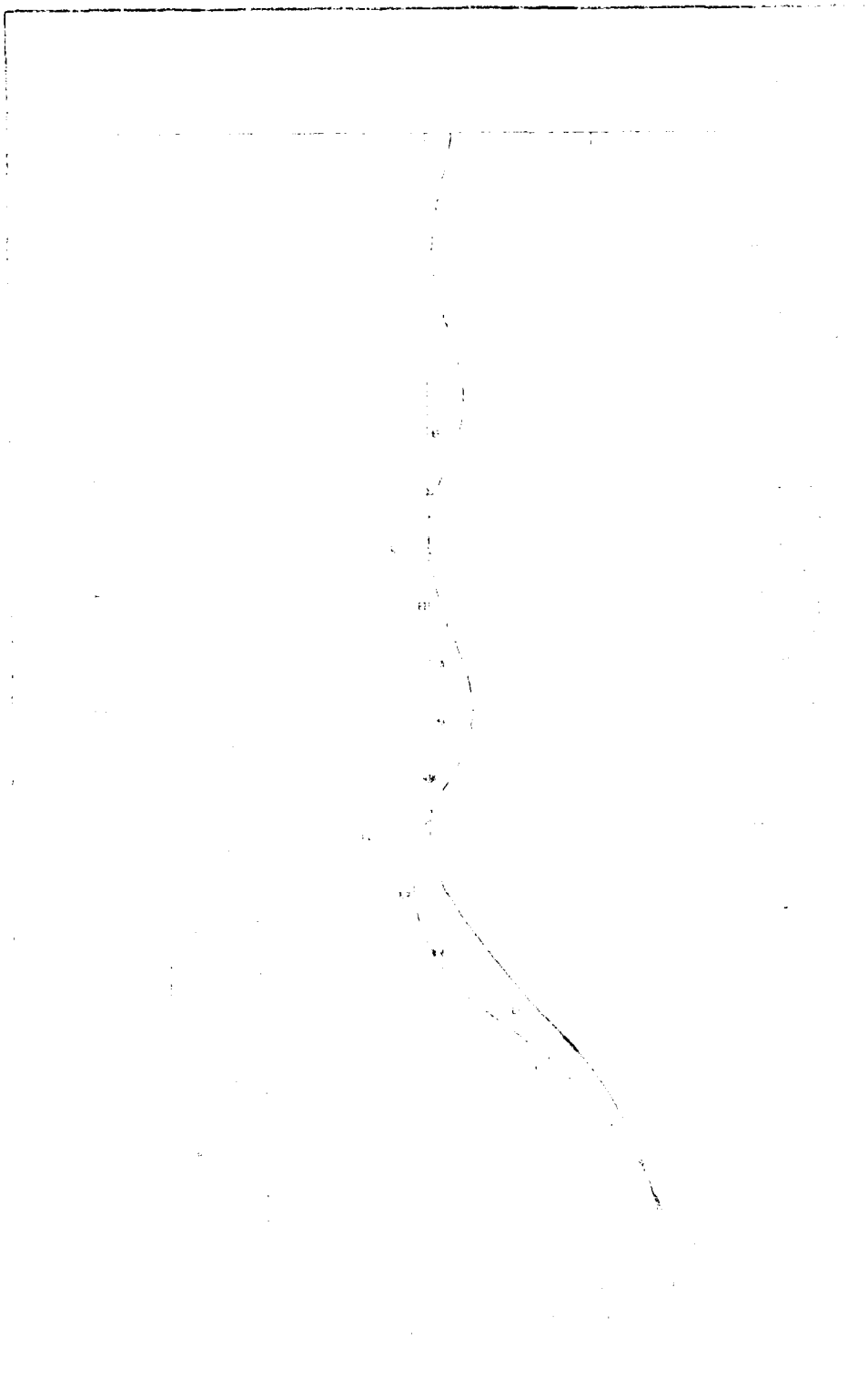


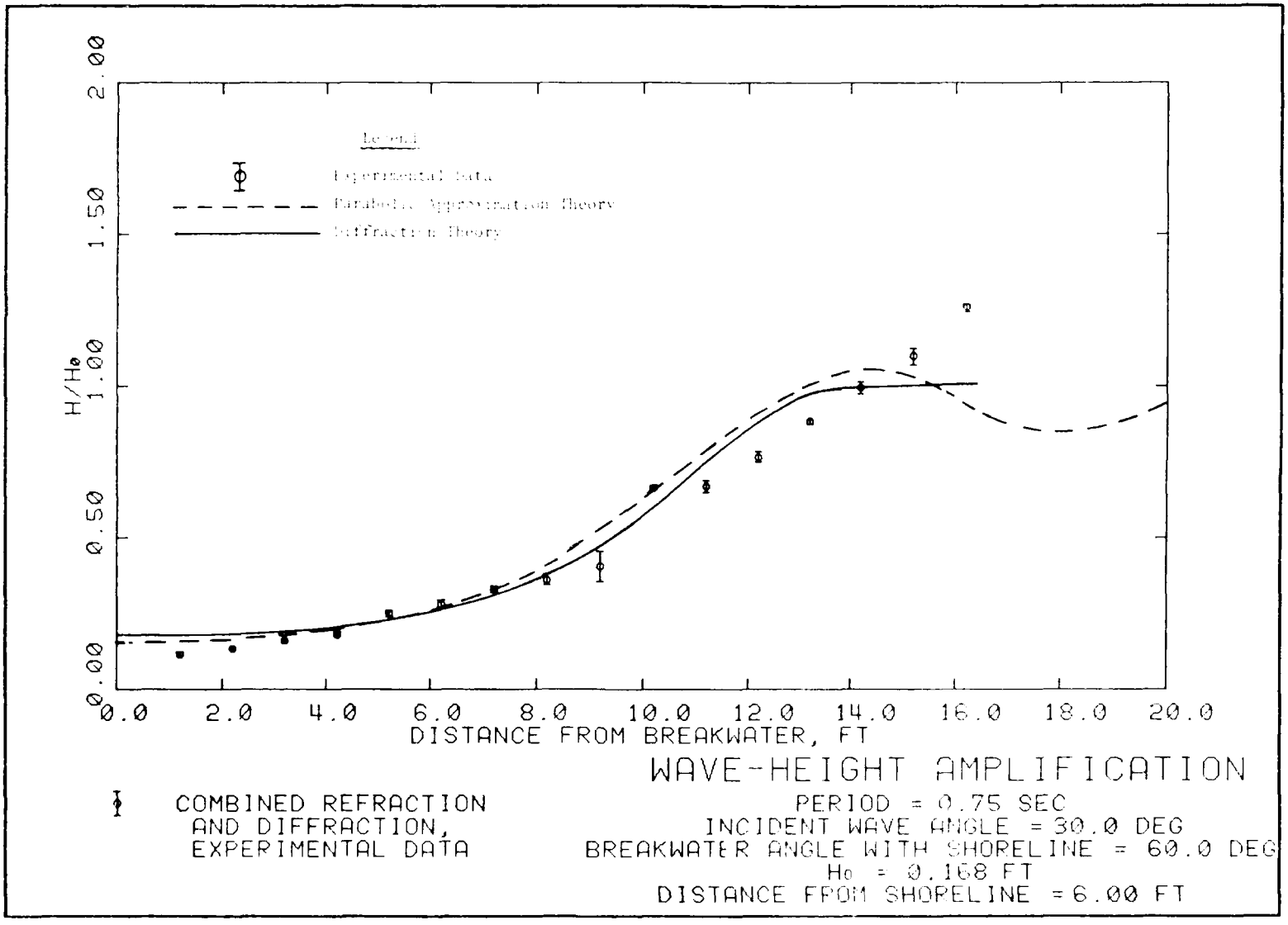
WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 0.75 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.106 FT
DISTANCE FROM SHORELINE = 10.00 FT

PLATE 6



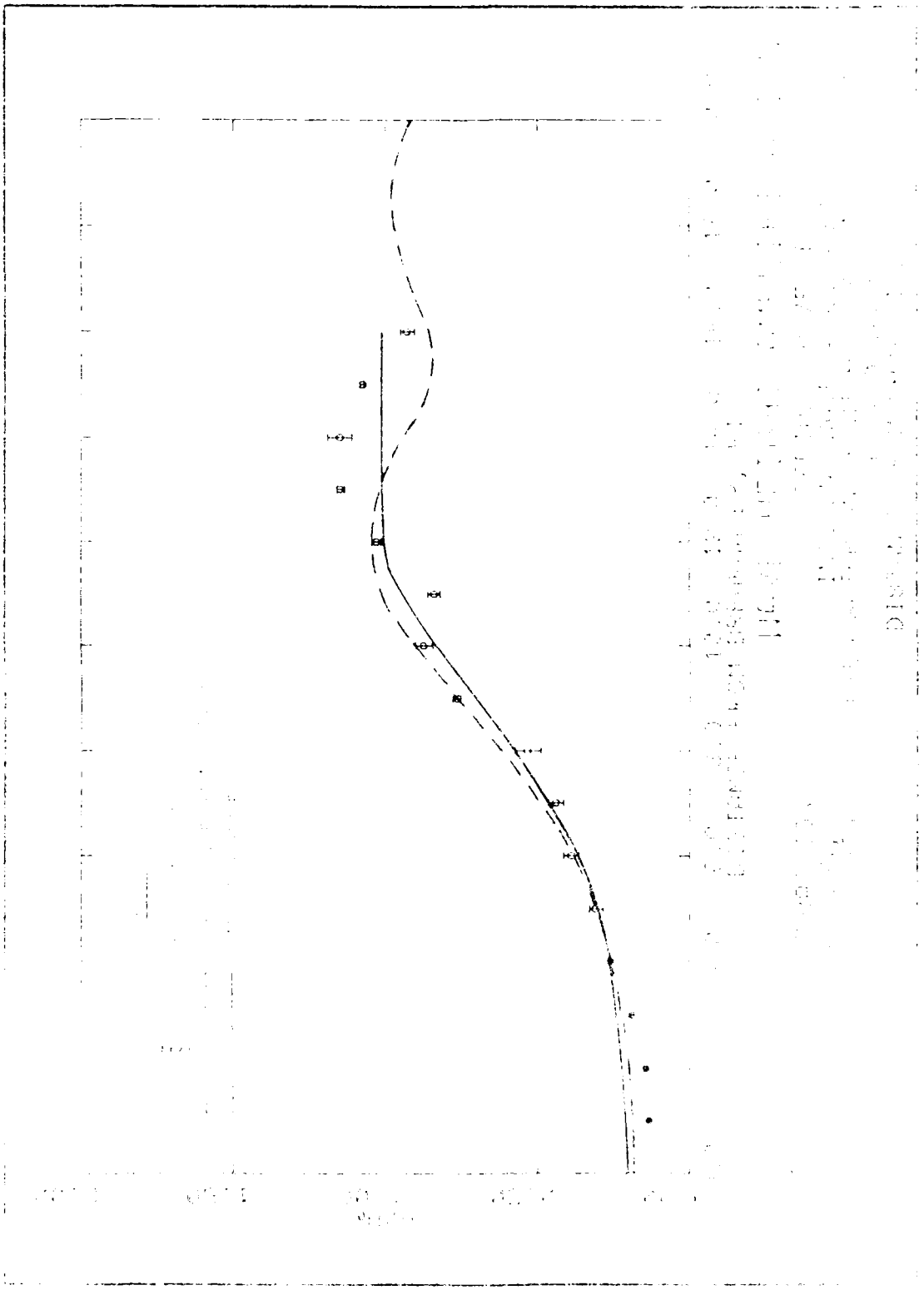


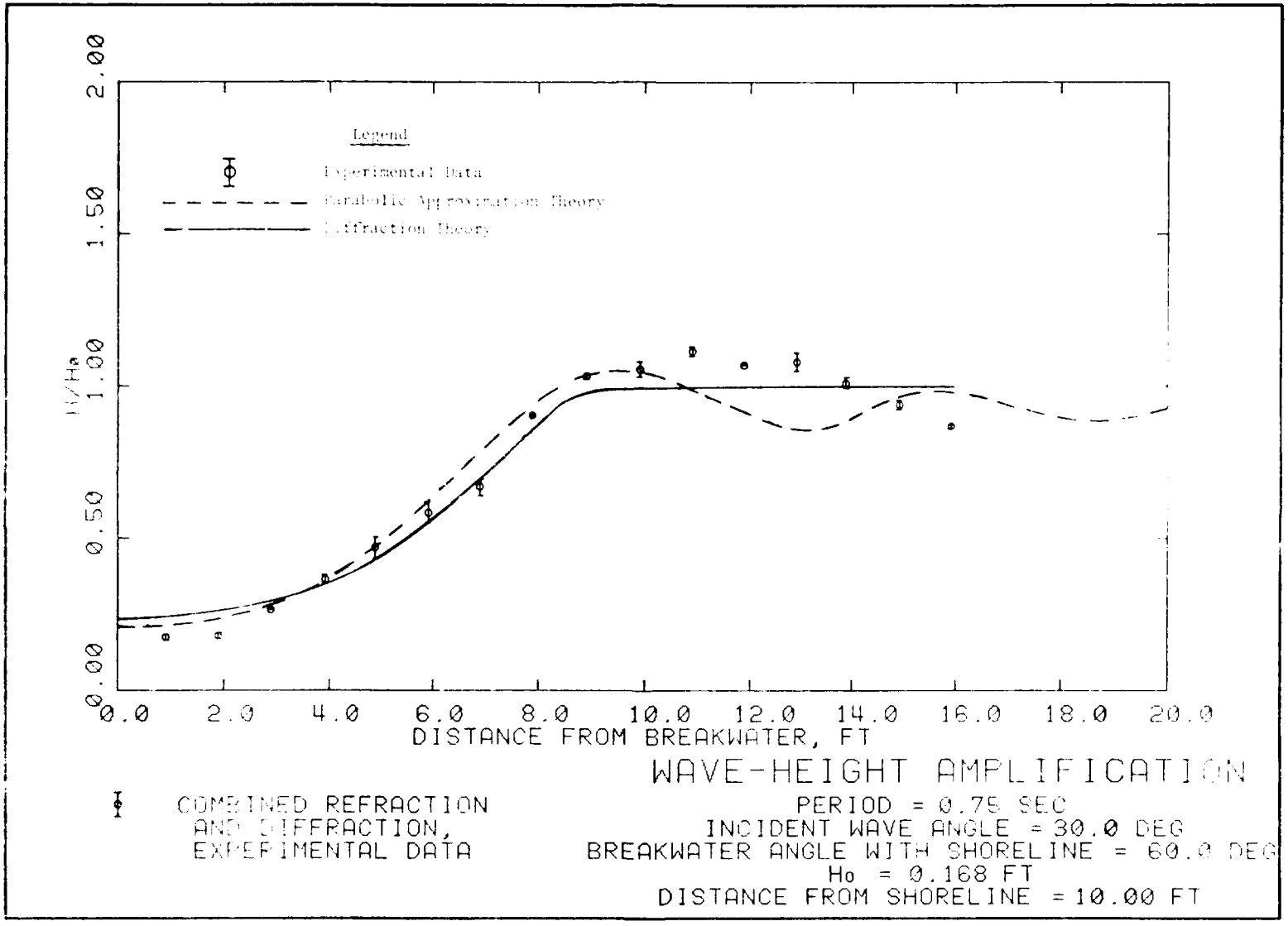
WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 0.75 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.168 FT
DISTANCE FROM SHORELINE = 6.00 FT

PLATE 6



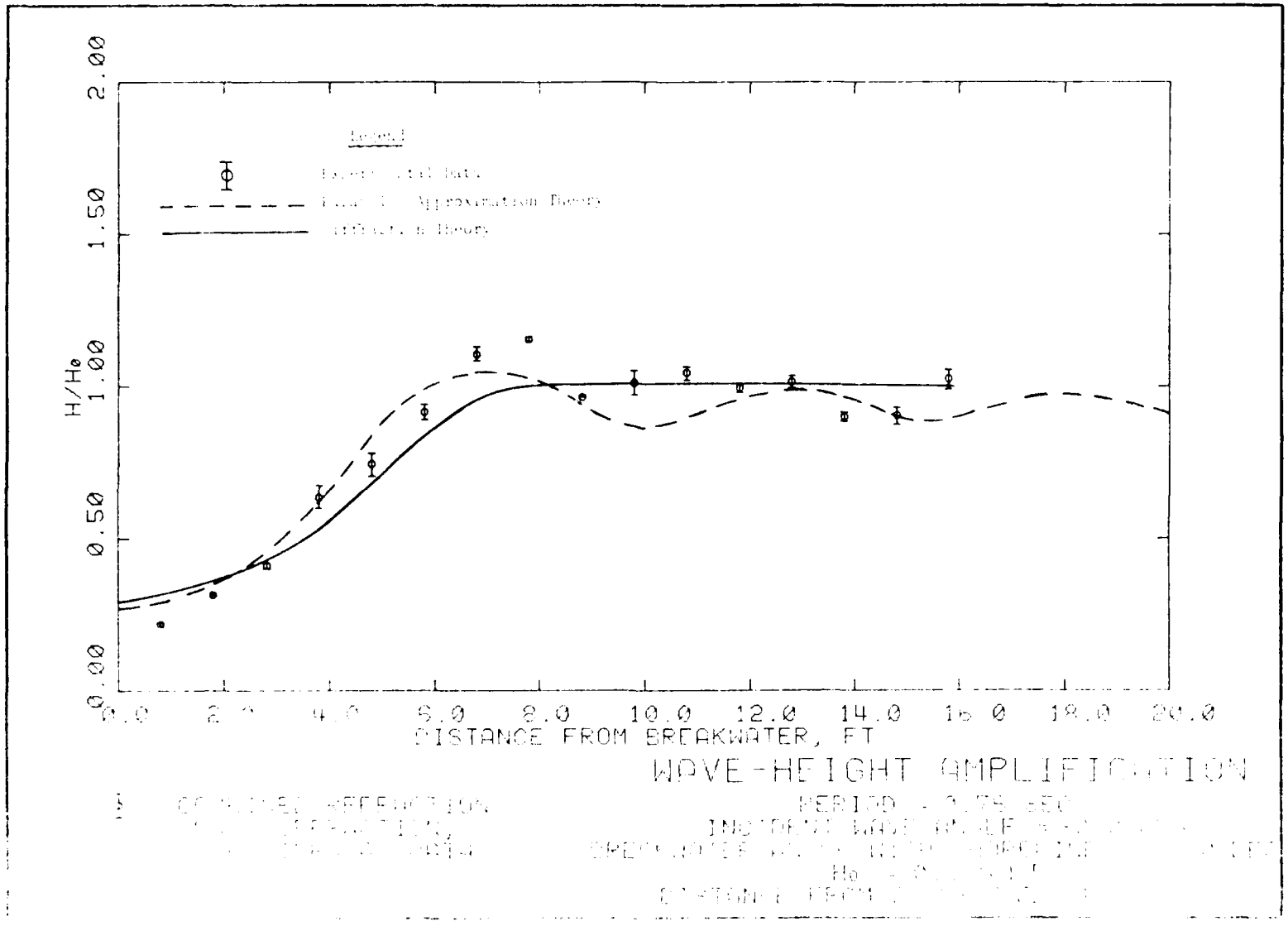


WAVE-HEIGHT AMPLIFICATION

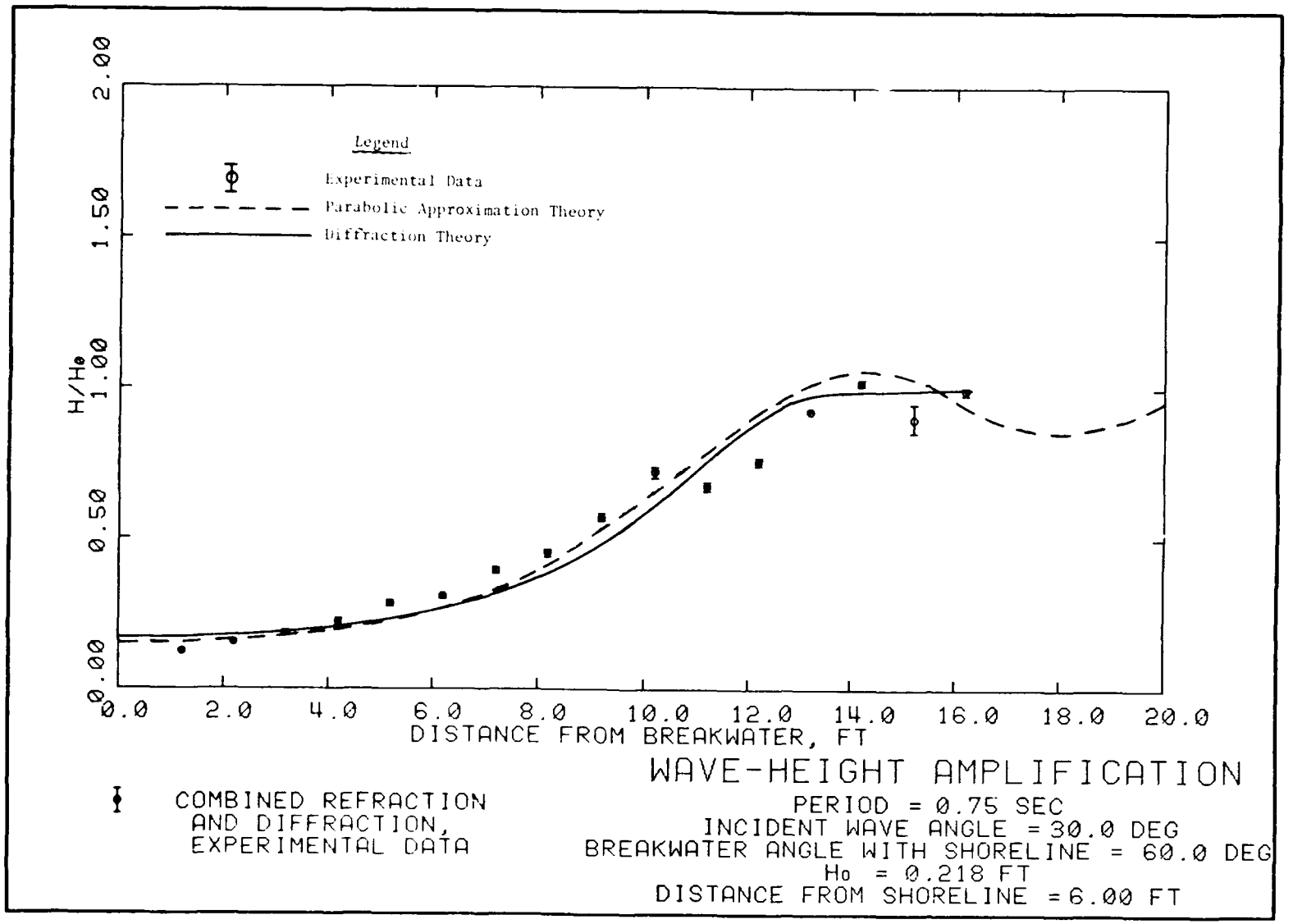
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 0.75 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.168 FT
DISTANCE FROM SHORELINE = 10.00 FT

PLATE 9



WAVE-HEIGHT AMPLIFICATION

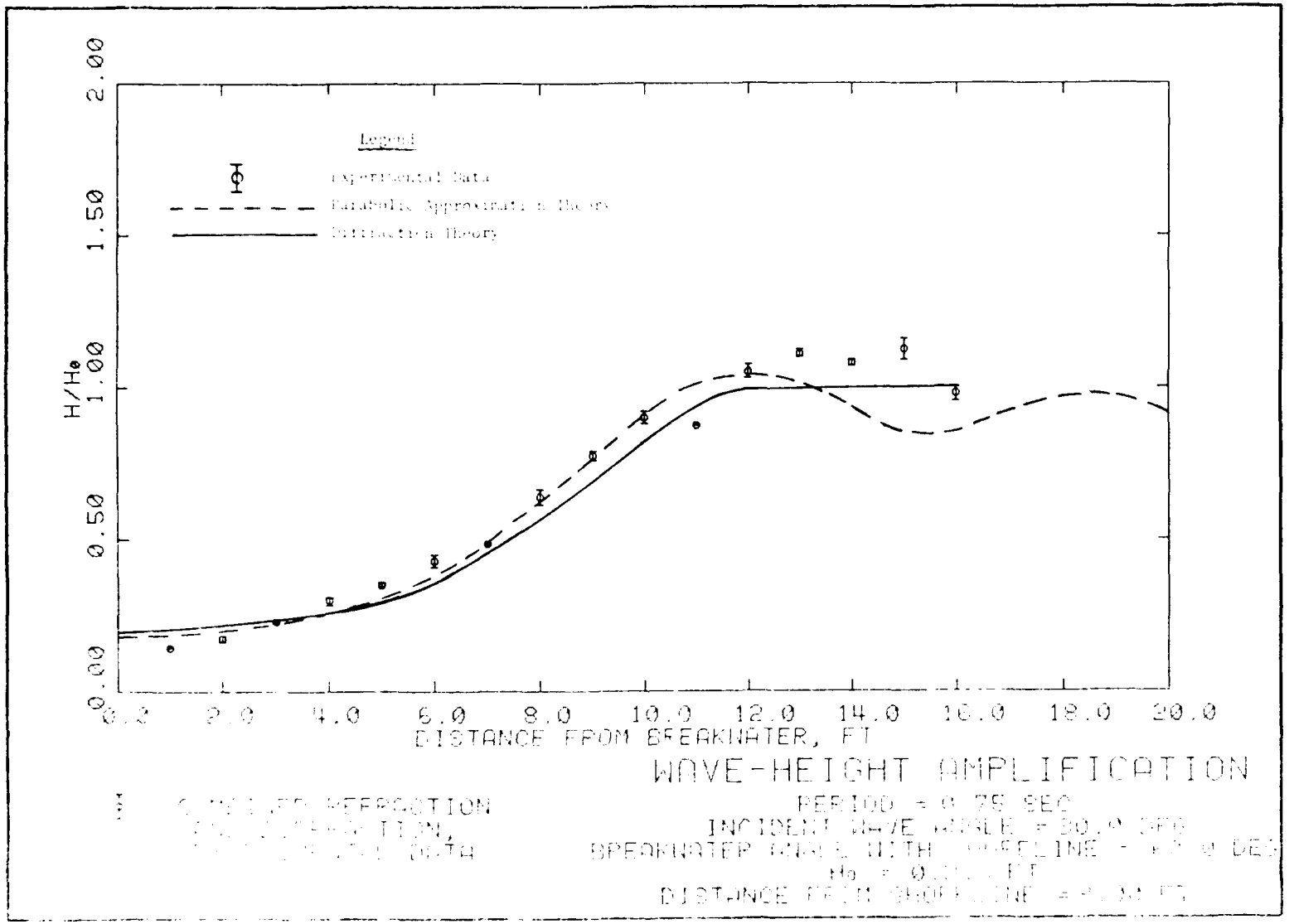


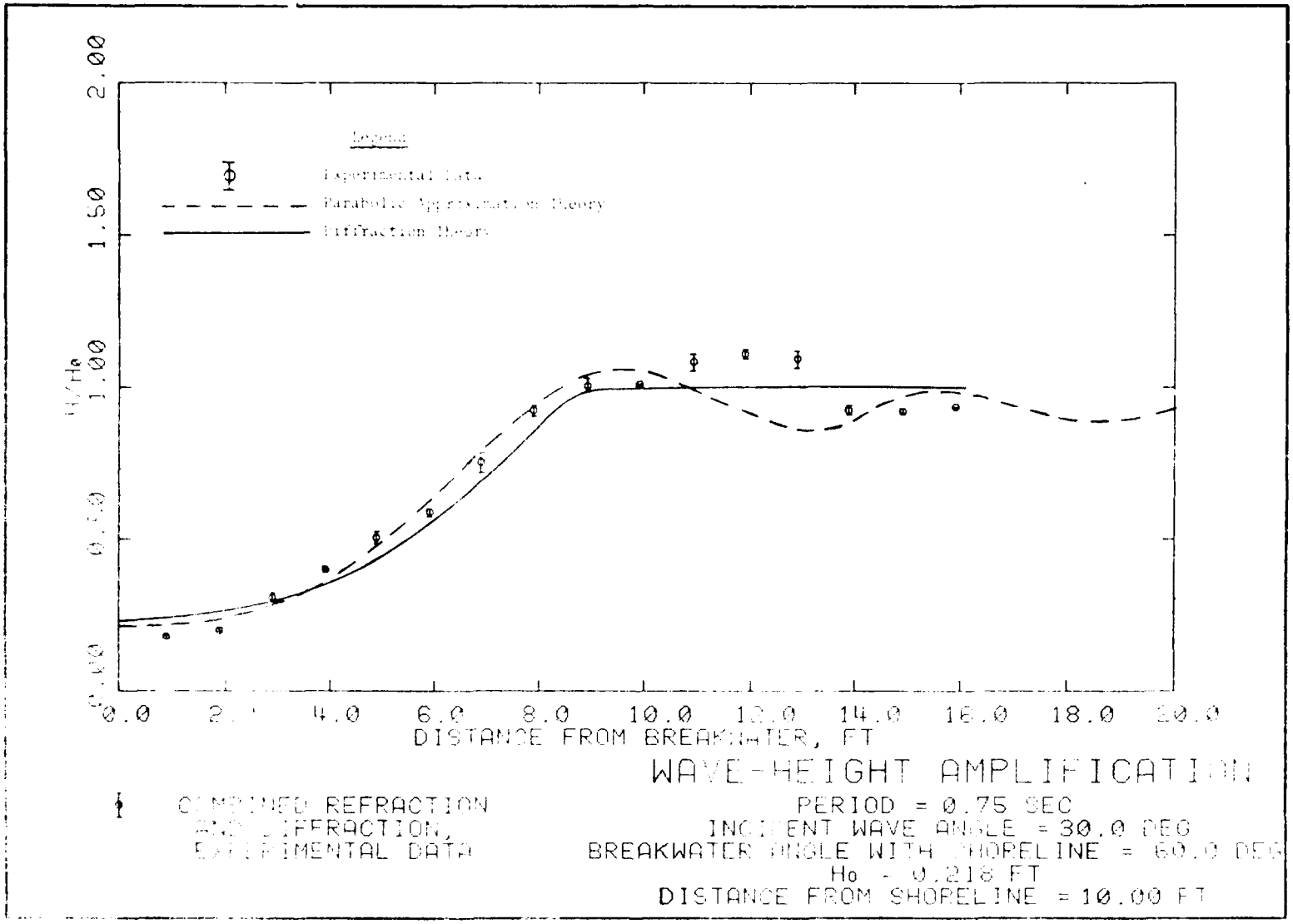
WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 0.75 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.218 FT
DISTANCE FROM SHORELINE = 6.00 FT

PLATE 9


Legend
Experimental Data
Parabolic Approximation Theory
Refraction Theory
H/Ho
2.00
1.50
1.00
0.50
0.00
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT
WAVE-HEIGHT AMPLIFICATION
PERIOD = 0.75 SEC


Legend
Experimental Data
Parabolic Approximation Theory
Diffraction Theory
2.00
1.50
1.00
H/Ho
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT
WAVE-HEIGHT AMPLIFICATION
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA
PERIOD = 0.75 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.218 FT
DISTANCE FROM SHORELINE = 10.00 FT

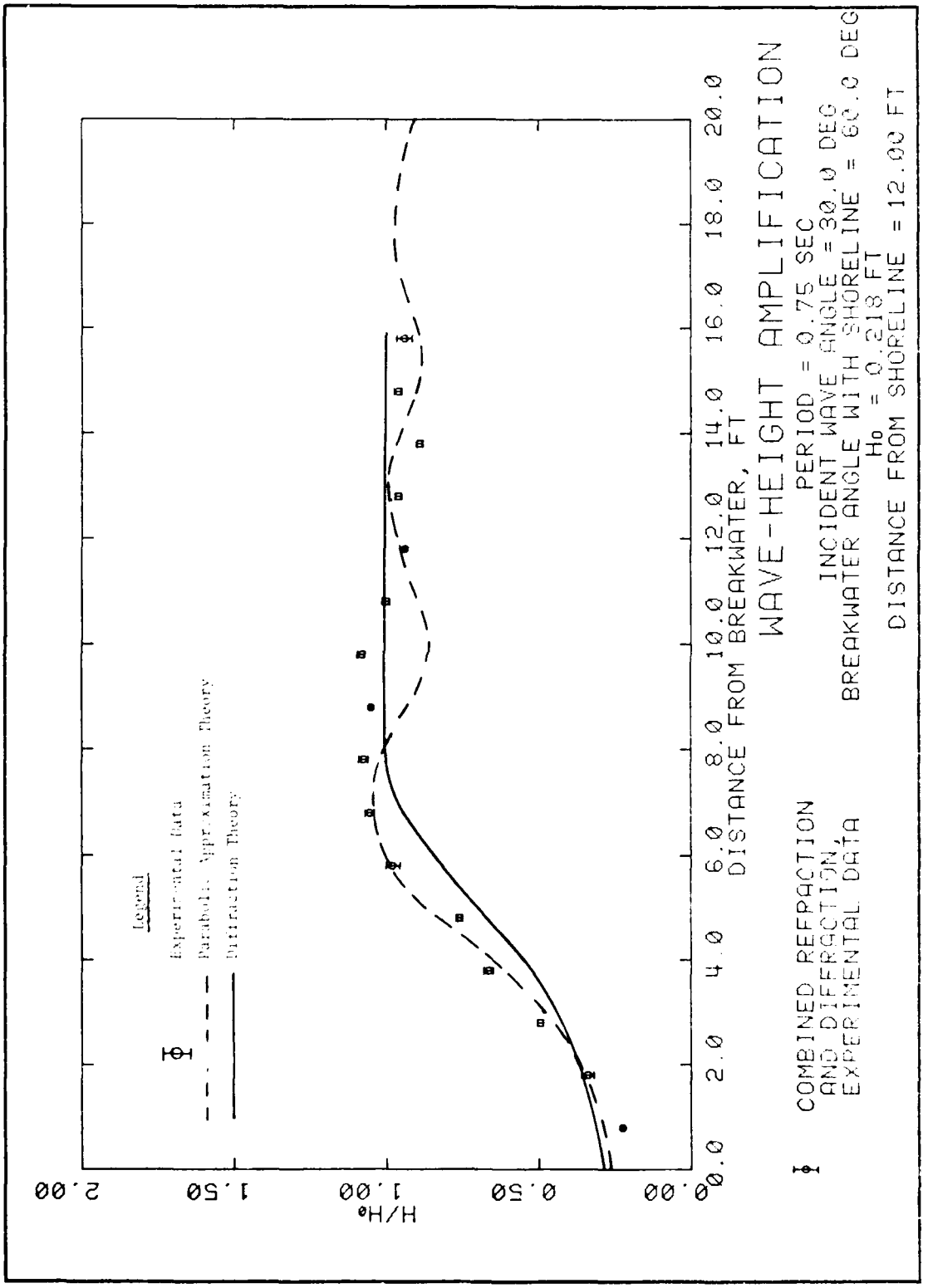
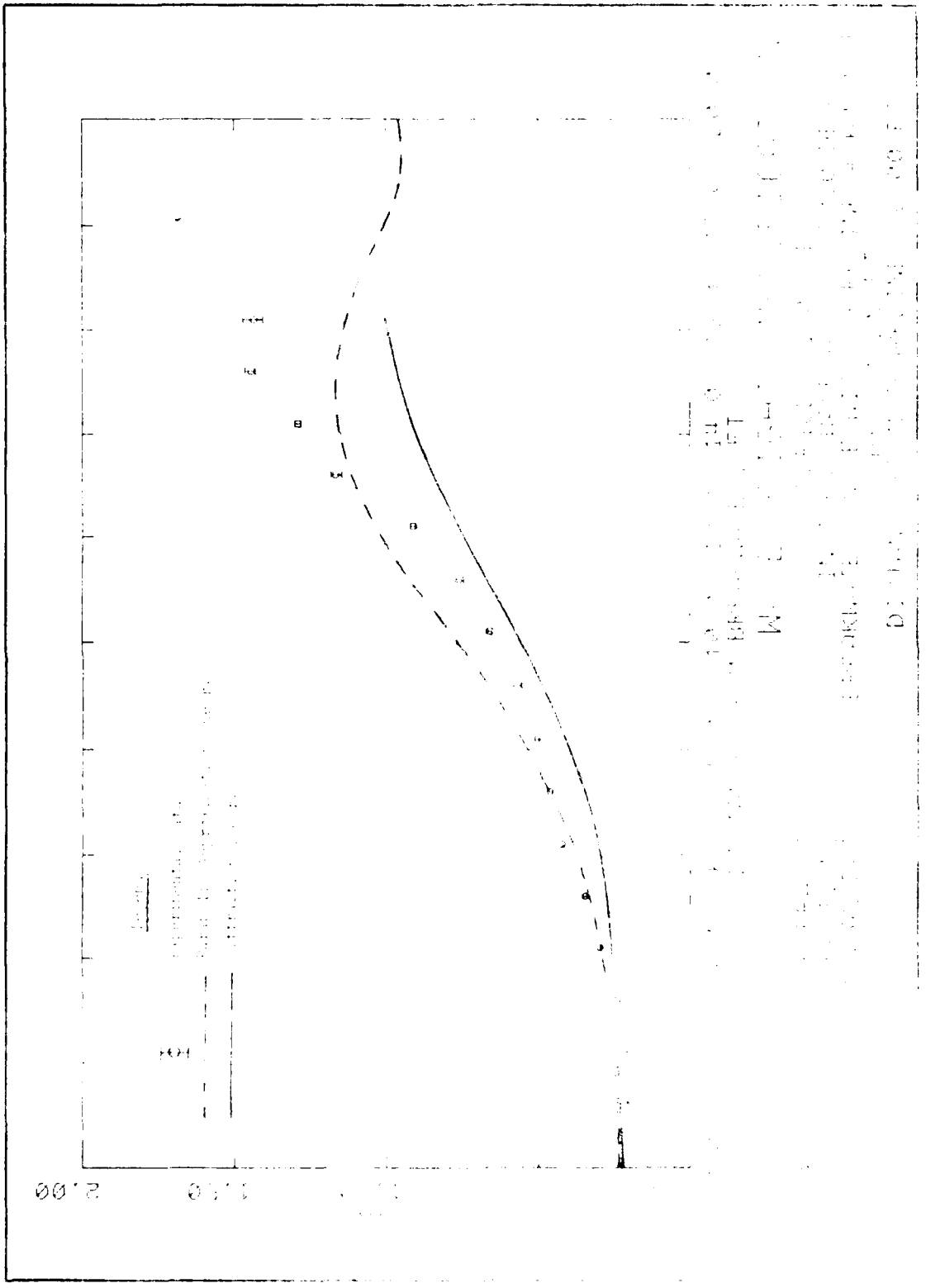
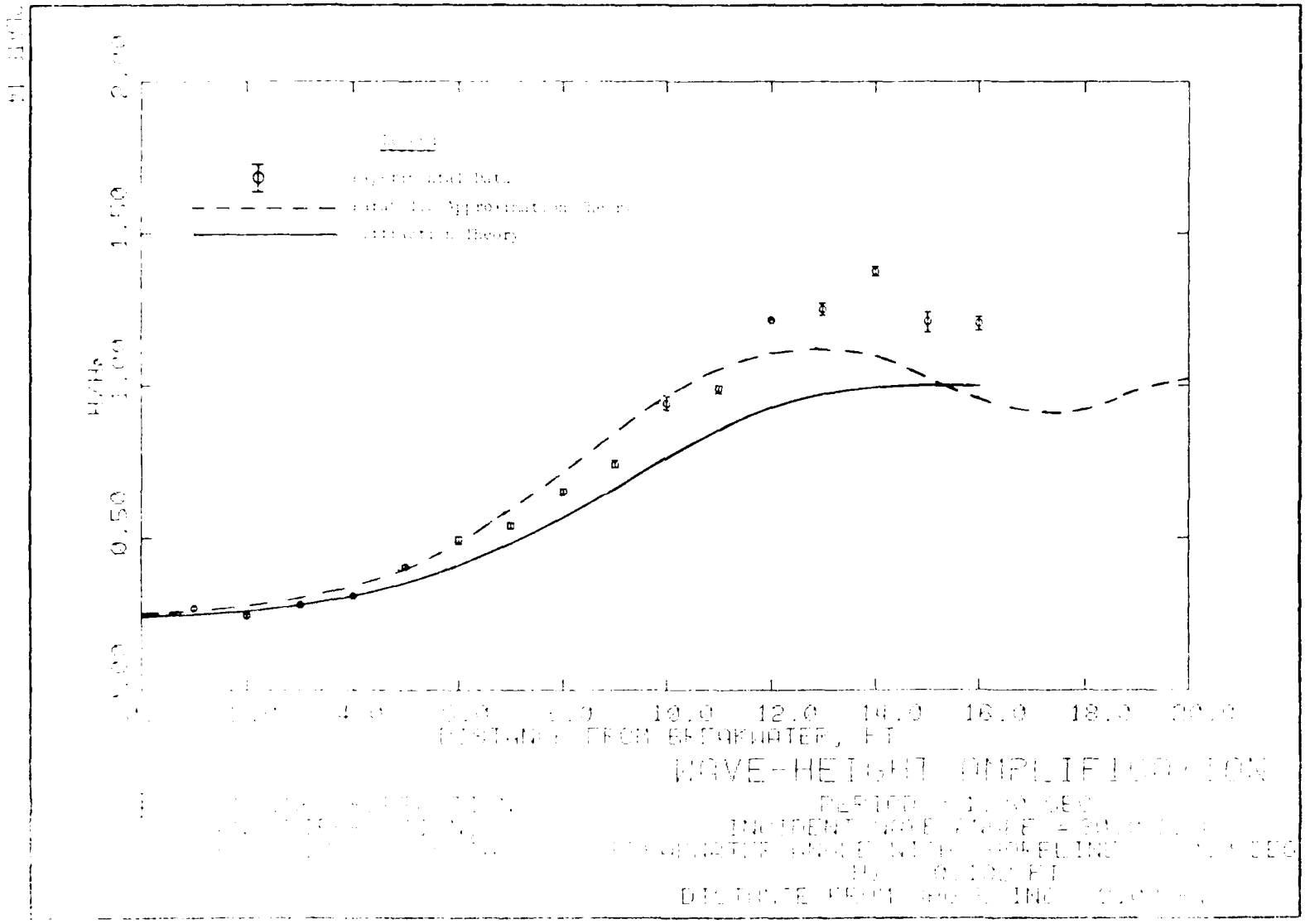
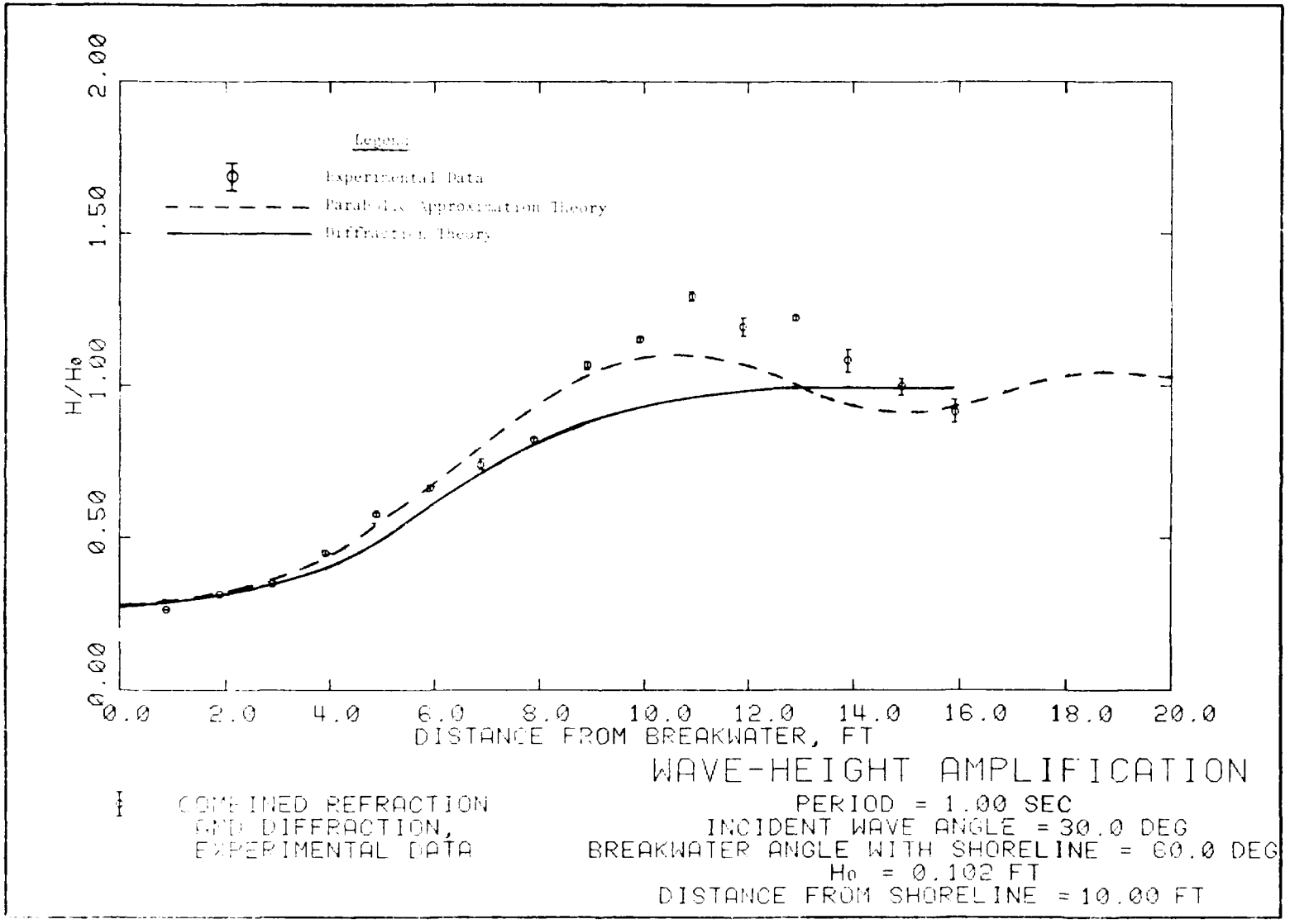
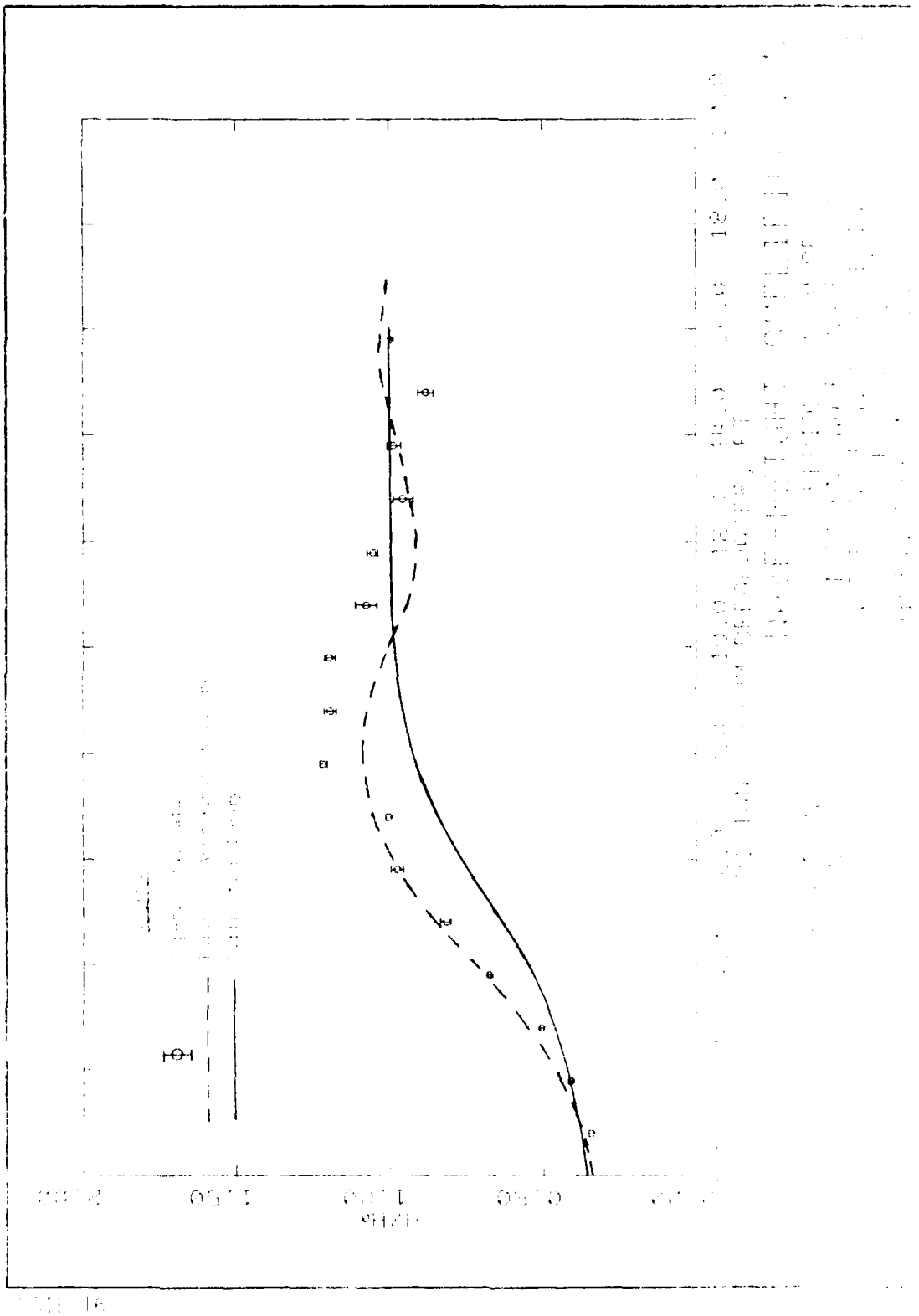

WAVE-HEIGHT AMPLIFICATION
PERIOD = 0.75 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.218 FT
DISTANCE FROM SHORELINE = 12.00 FT
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA
DISTANCE FROM BREAKWATER, FT
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
H/Ho
0.00
0.50
1.00
1.50
2.00
Legend
Experimental Data
Parabolic Approximation Theory
Diffraction Theory

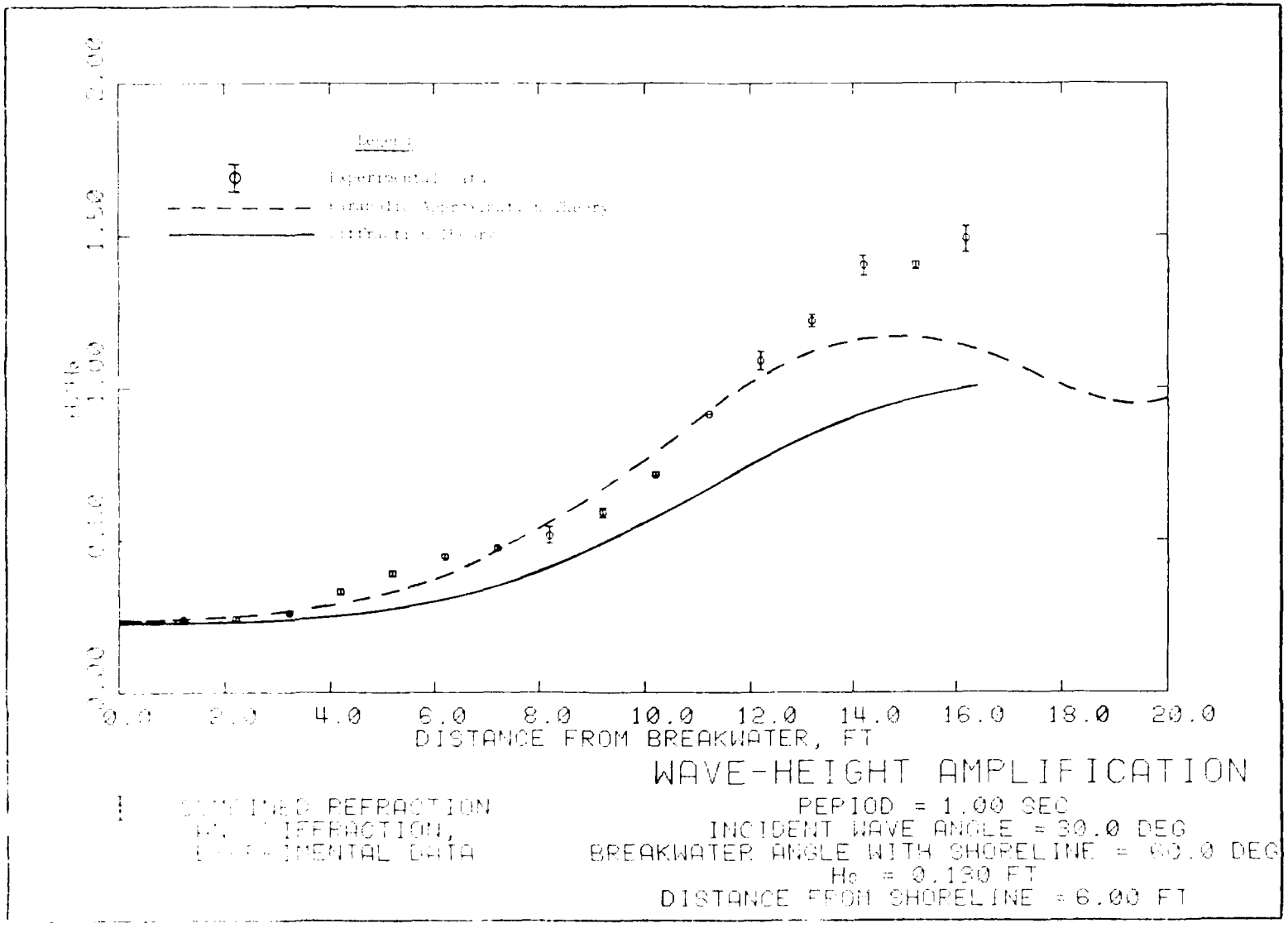
PLATE 12

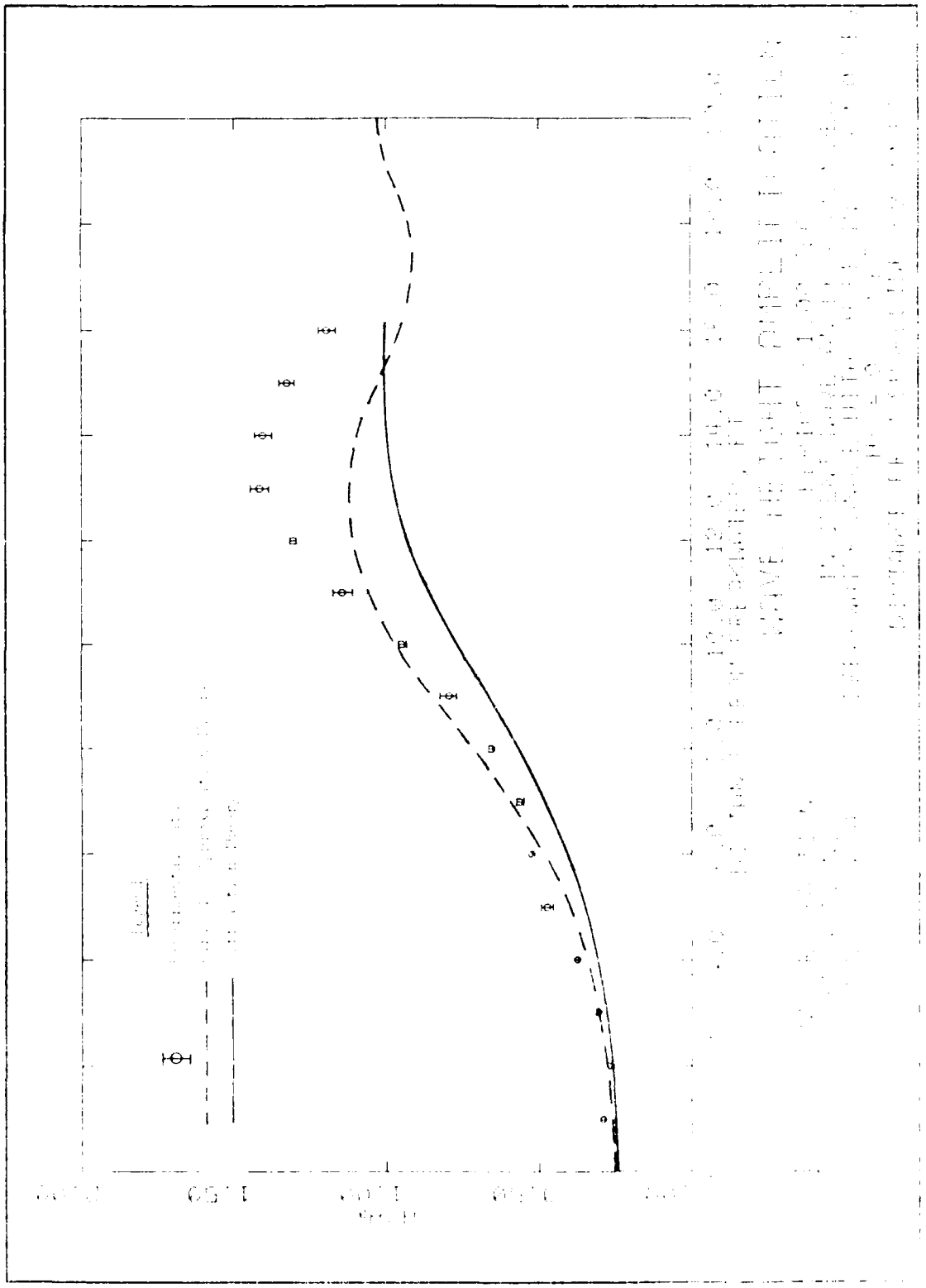



1.50
0.50
10.0
12.0
DISTANCE FROM BREAKWATER, FT

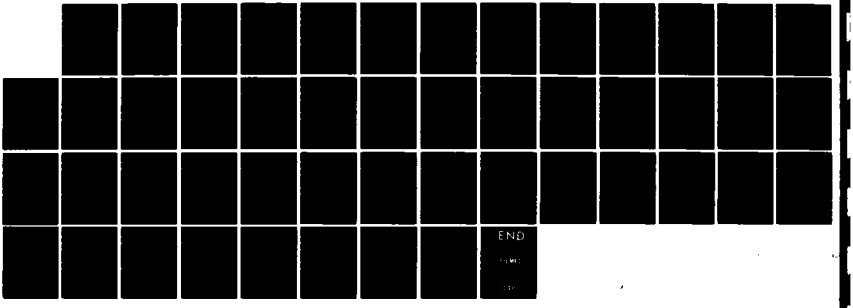

Legend:
Experimental Data
Parabolic Approximation Theory
Diffraction Theory
H/H₀
2.00
1.50
1.00
0.50
0.00
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT
WAVE-HEIGHT AMPLIFICATION
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA
PERIOD = 1.00 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H₀ = 0.102 FT
DISTANCE FROM SHORELINE = 10.00 FT

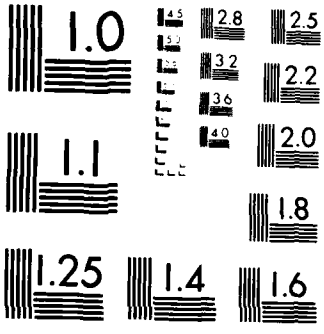


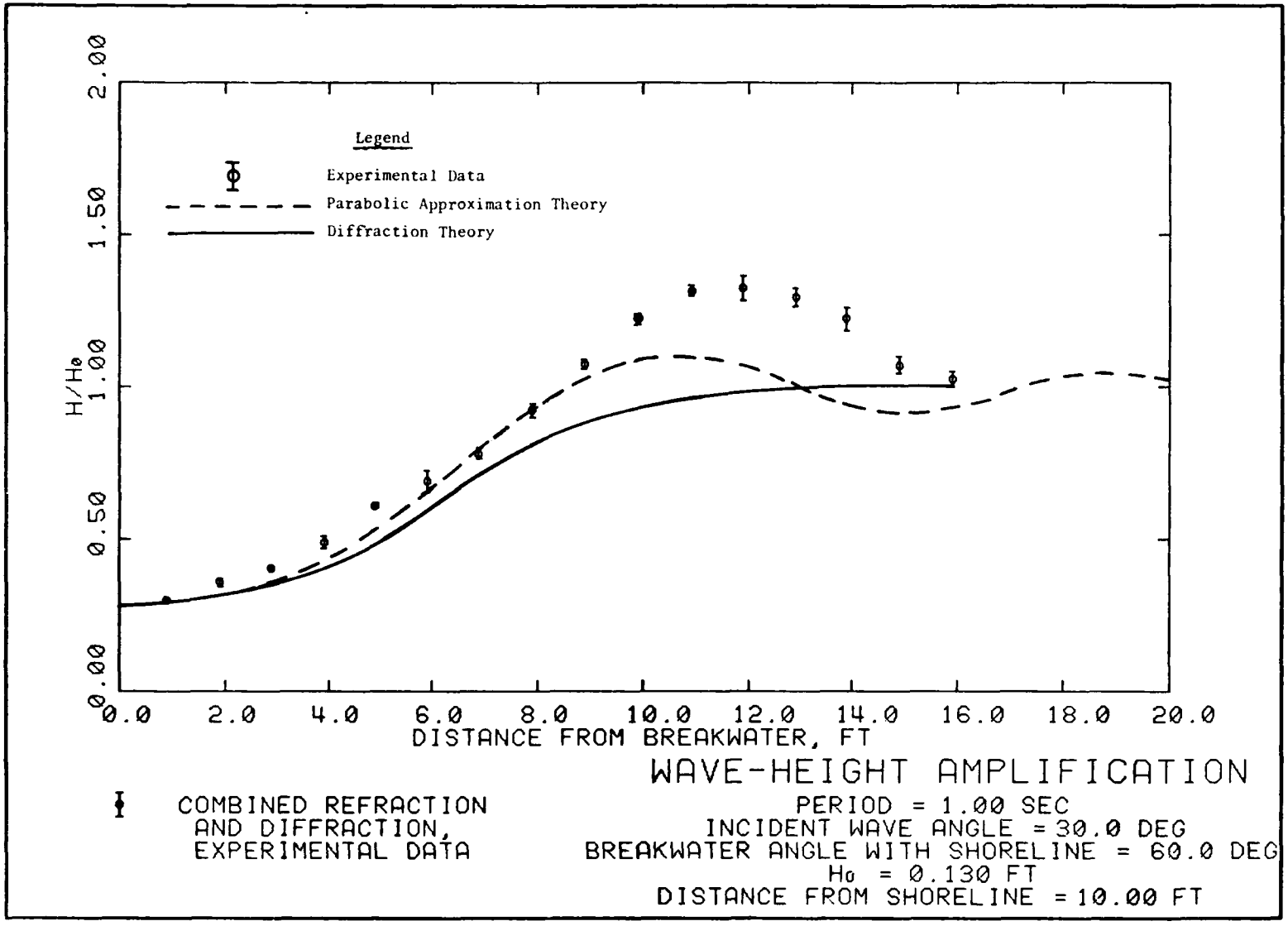

4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT
WAVE-HEIGHT AMPLIFICATION
PERIOD = 1.00 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 90.0 DEG
Hs = 0.130 FT
DISTANCE FROM SHORELINE = 6.00 FT



AD-A149 743 EROSION CONTROL OF SCOUR DURING CONSTRUCTION REPORT 5 2/2
EXPERIMENTAL MEASUR..(U) ARMY ENGINEER WATERWAYS
EXPERIMENT STATION VICKSBURG MS HYDRA.. L Z HALES
UNCLASSIFIED SEP 84 WES/TR/HL-80-3-5 F/G 13/2 NL







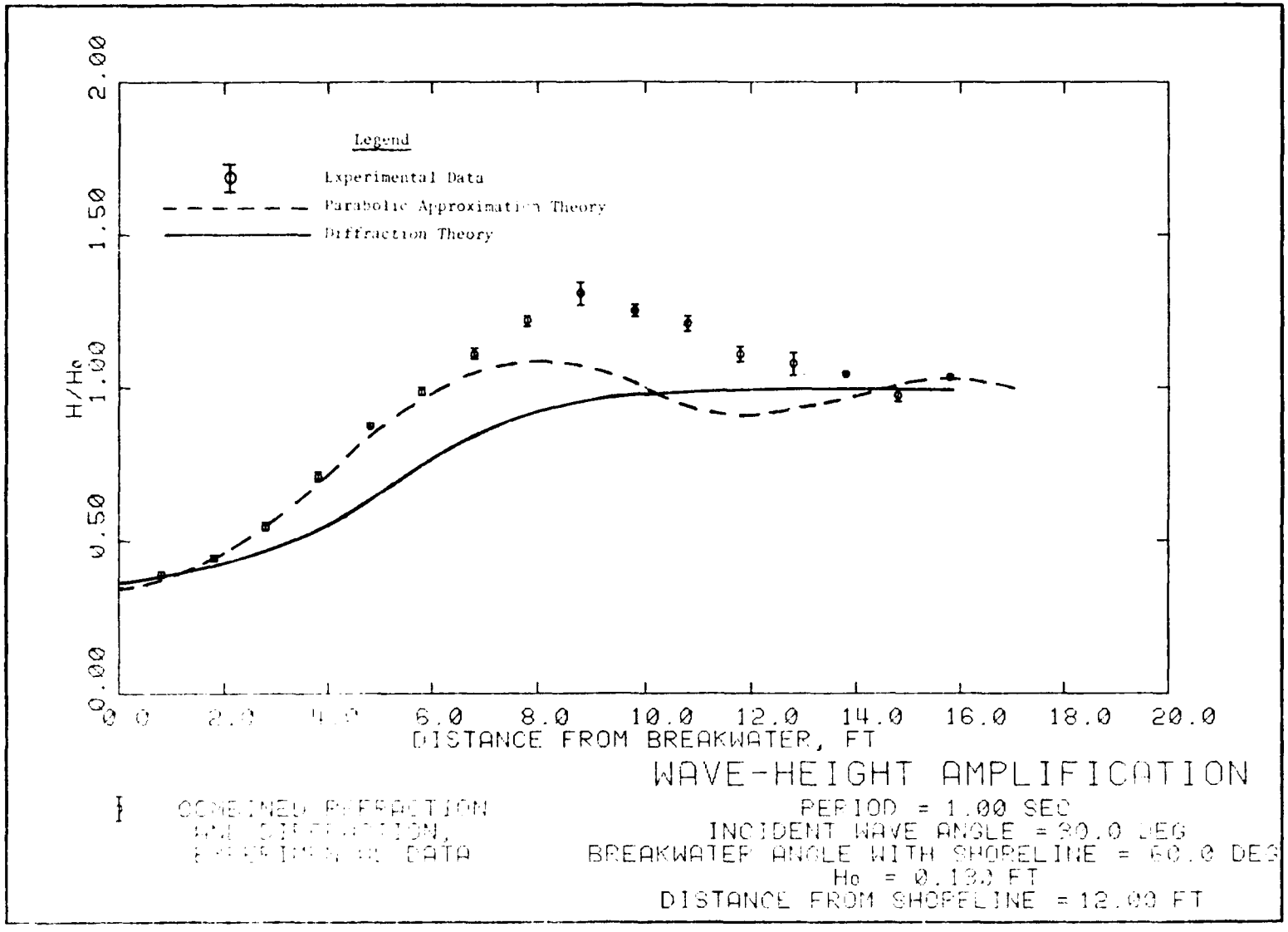
WAVE-HEIGHT AMPLIFICATION

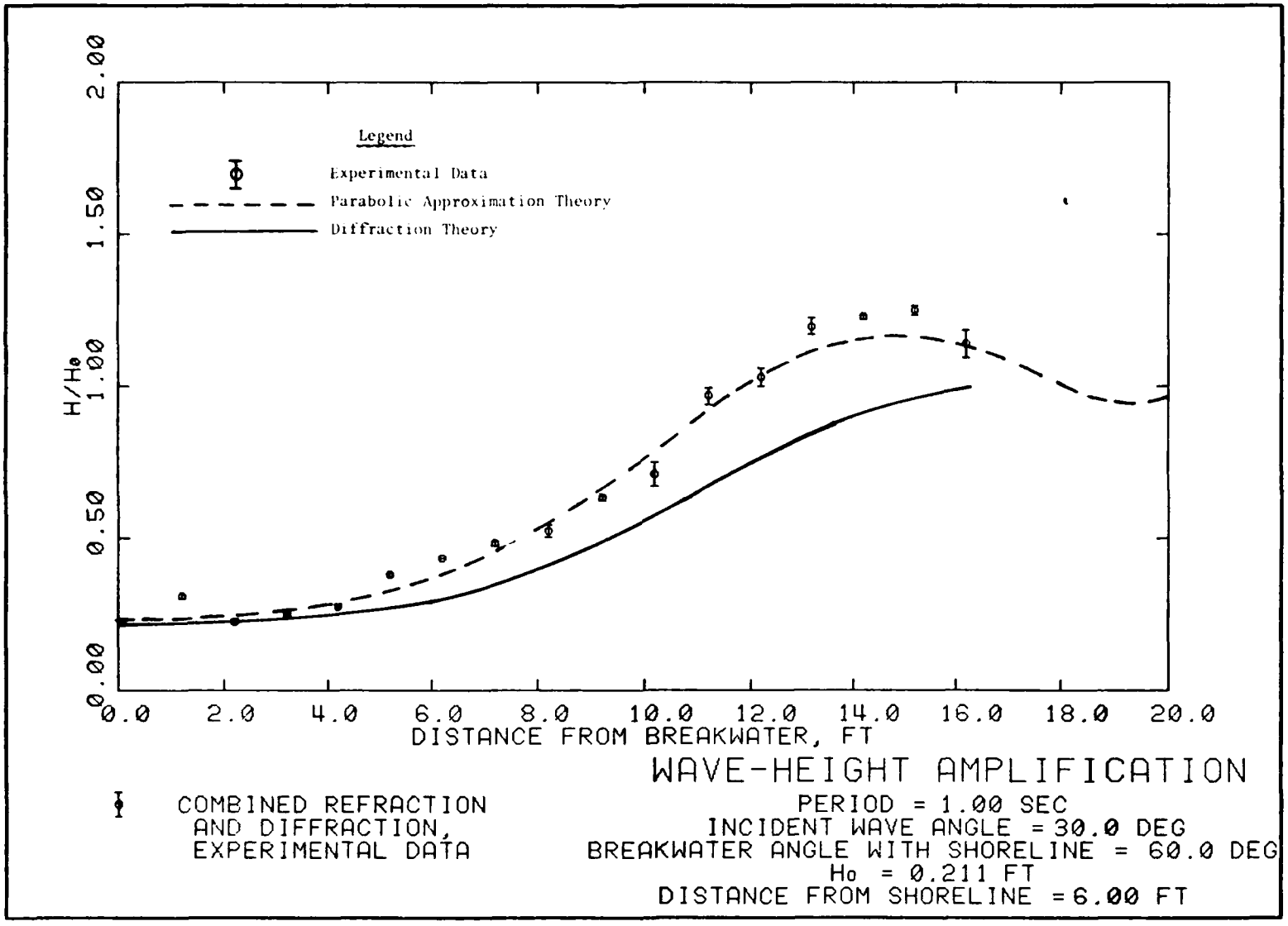
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.00 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.130 FT
DISTANCE FROM SHORELINE = 10.00 FT

PLATE 19

PLATE 20


Legend
Experimental Data
Parabolic Approximation Theory
Diffraction Theory
H/Ho
0.00
0.50
1.00
1.50
2.00
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA
WAVE-HEIGHT AMPLIFICATION
PERIOD = 1.00 SEC
INCIDENT WAVE ANGLE = 90.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.130 FT
DISTANCE FROM SHORELINE = 12.00 FT



WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.00 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.211 FT
DISTANCE FROM SHORELINE = 6.00 FT

PLATE 21

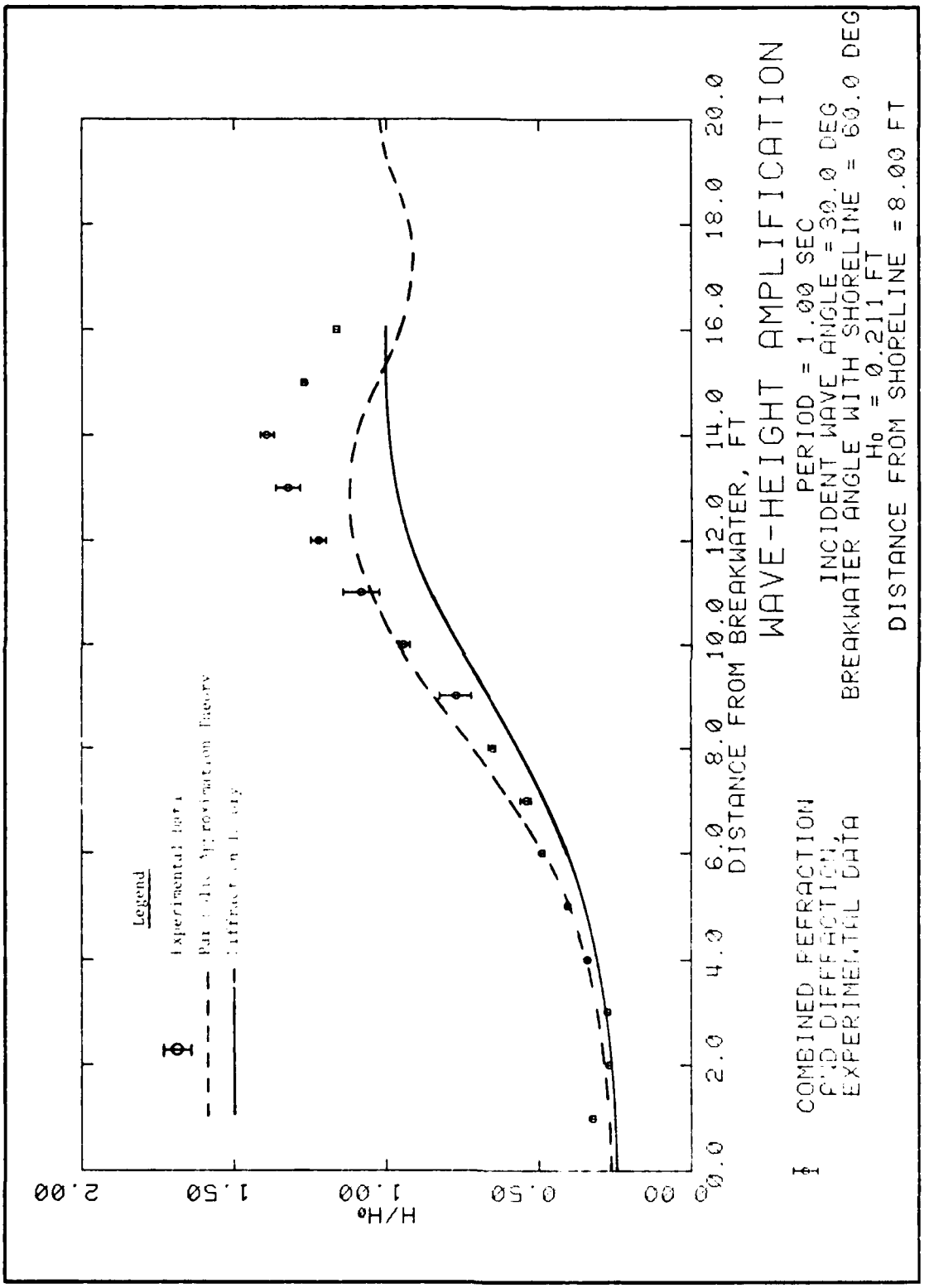
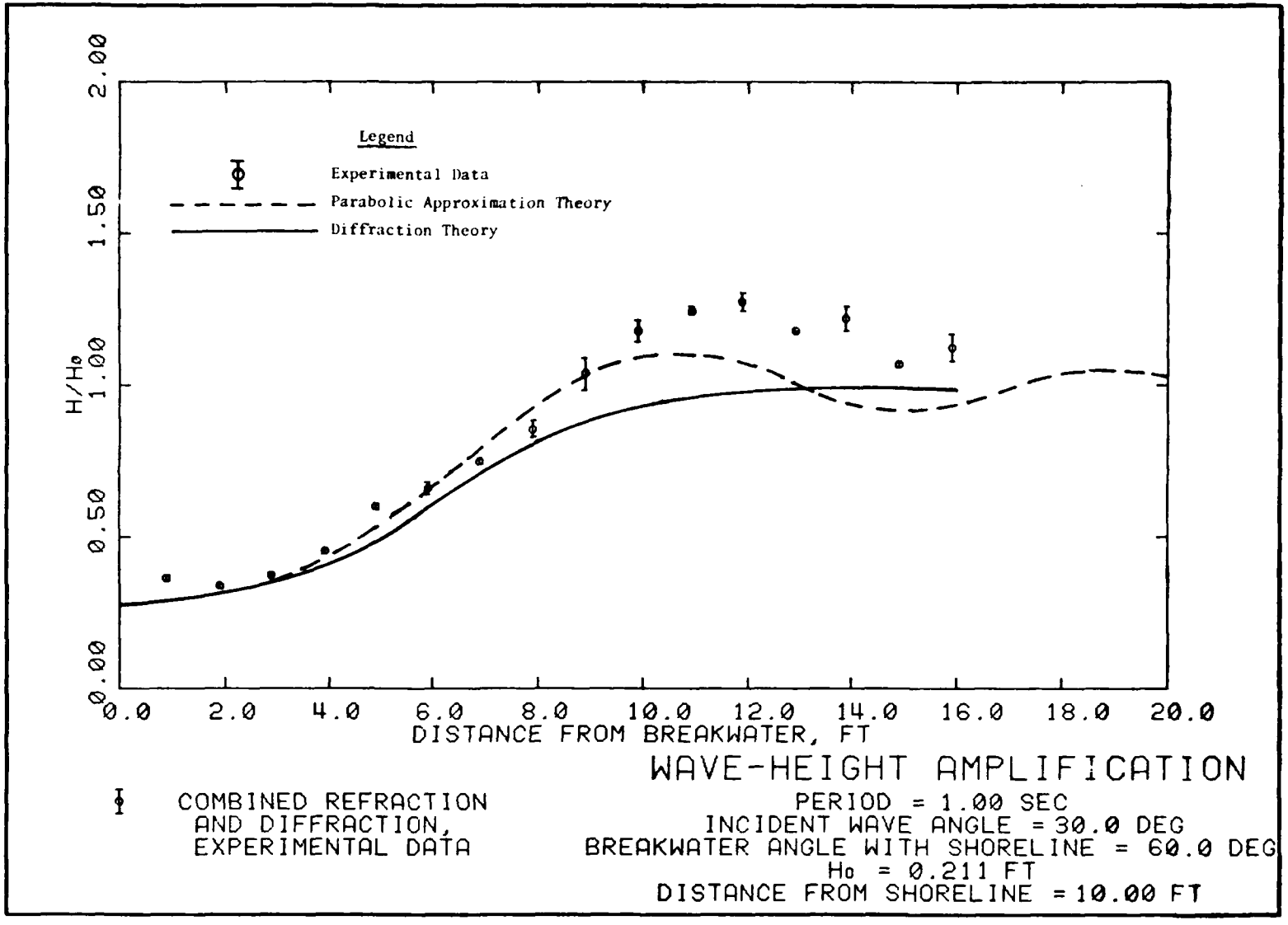

WAVE-HEIGHT AMPLIFICATION
PERIOD = 1.00 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.211 FT
DISTANCE FROM SHORELINE = 8.00 FT
COMBINED REFRACTION
AND DIFFRACTION,
EXPERIMENTAL DATA
Legend
Experimental data
Diffraction theory
H/H$_o$
0.00
0.50
1.00
1.50
2.00
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT

PLATE 22



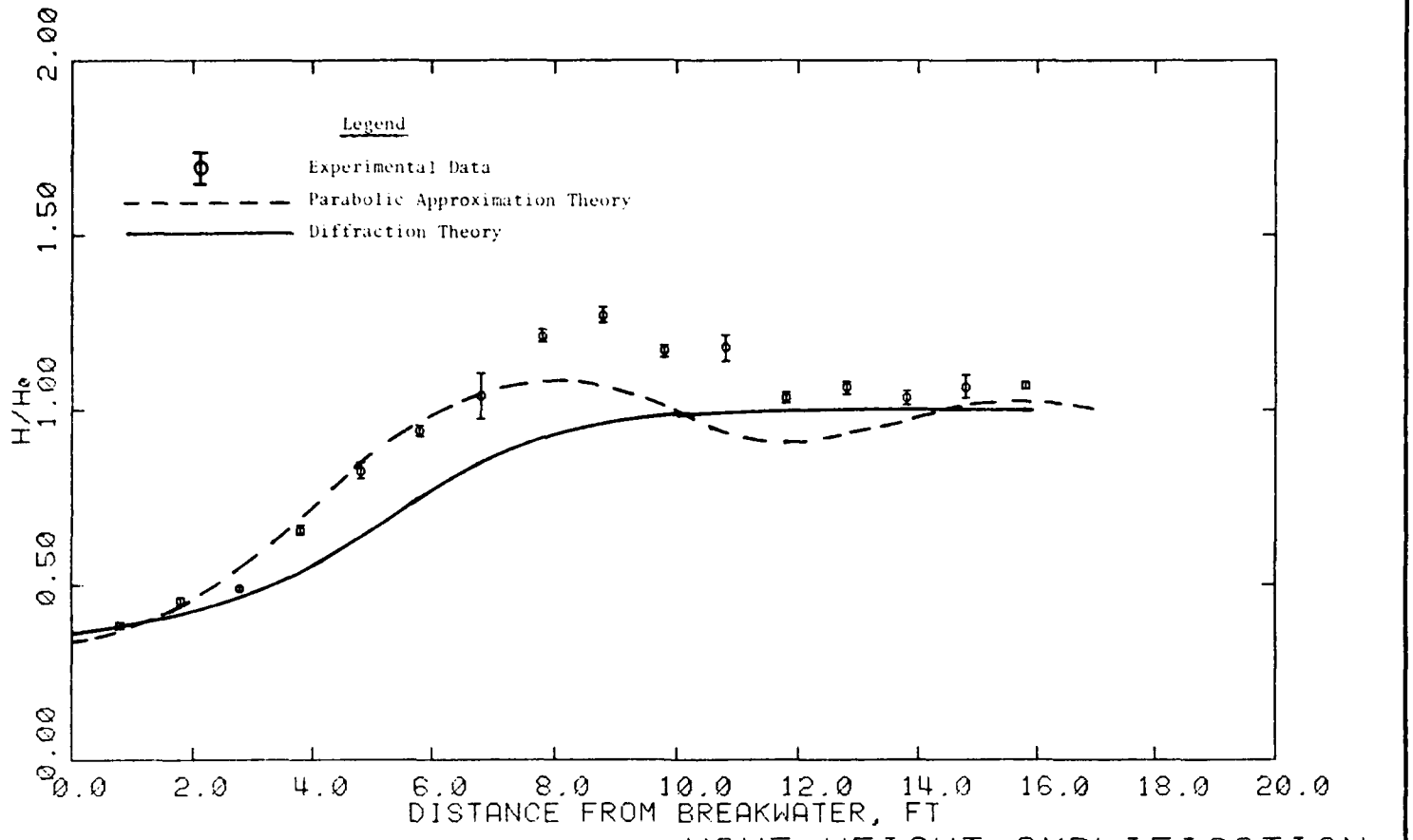
WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.00 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.211 FT
DISTANCE FROM SHORELINE = 10.00 FT

PLATE 23

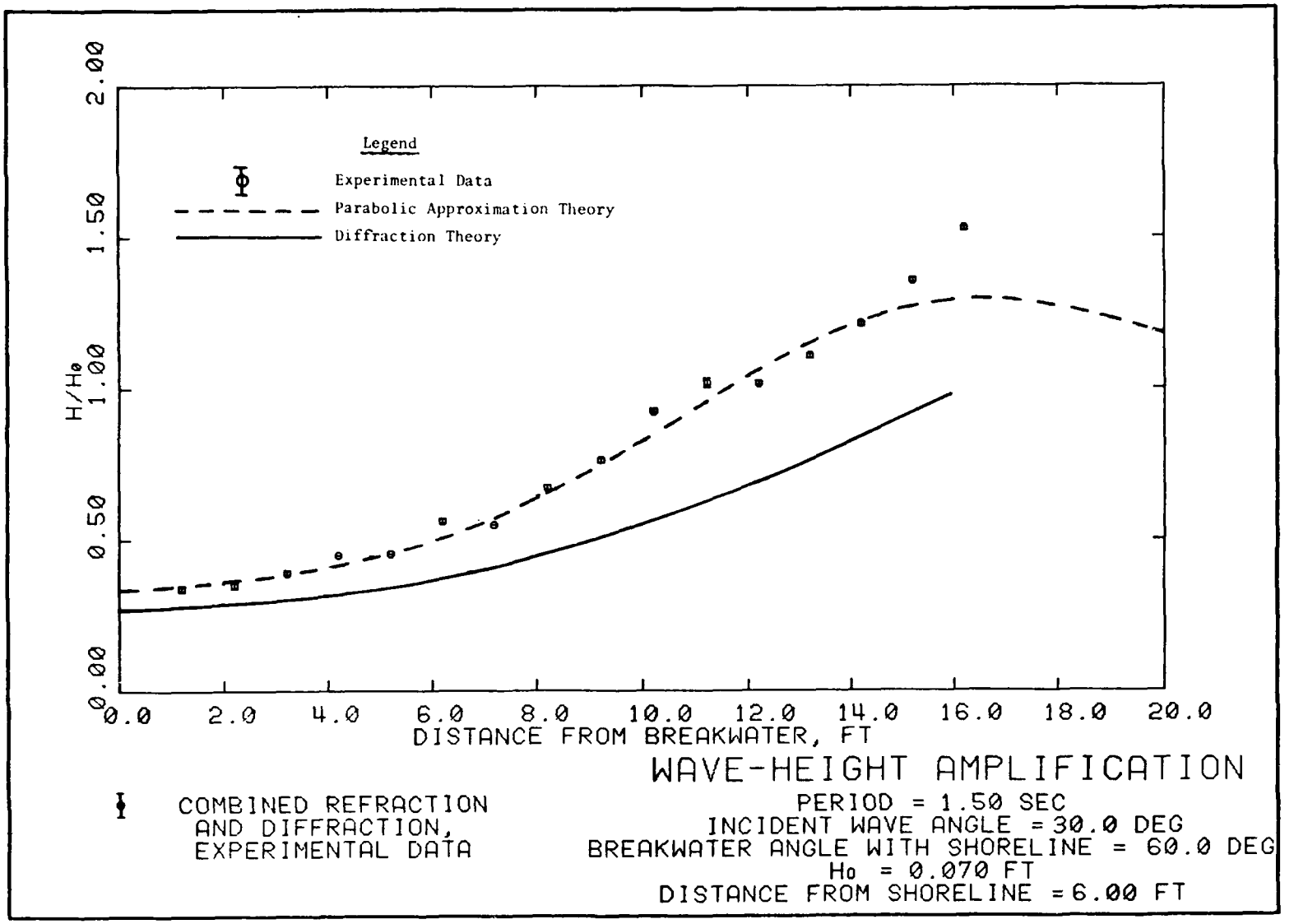
PLATE 24



WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.00 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.211 FT
DISTANCE FROM SHORELINE = 12.00 FT



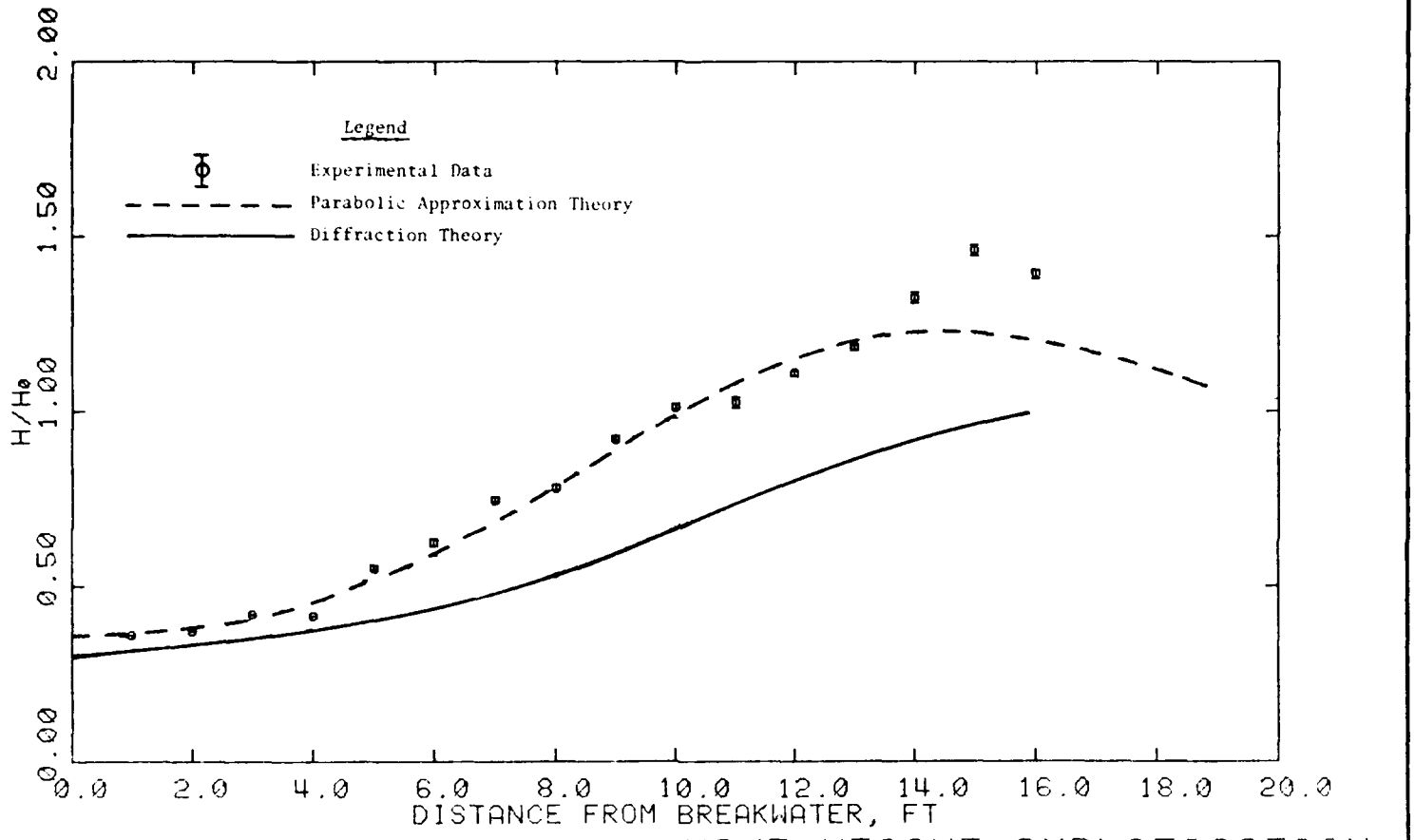
WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.070 FT
DISTANCE FROM SHORELINE = 6.00 FT

PLATE 25

PLATE 26



WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.070 FT
DISTANCE FROM SHORELINE = 8.00 FT

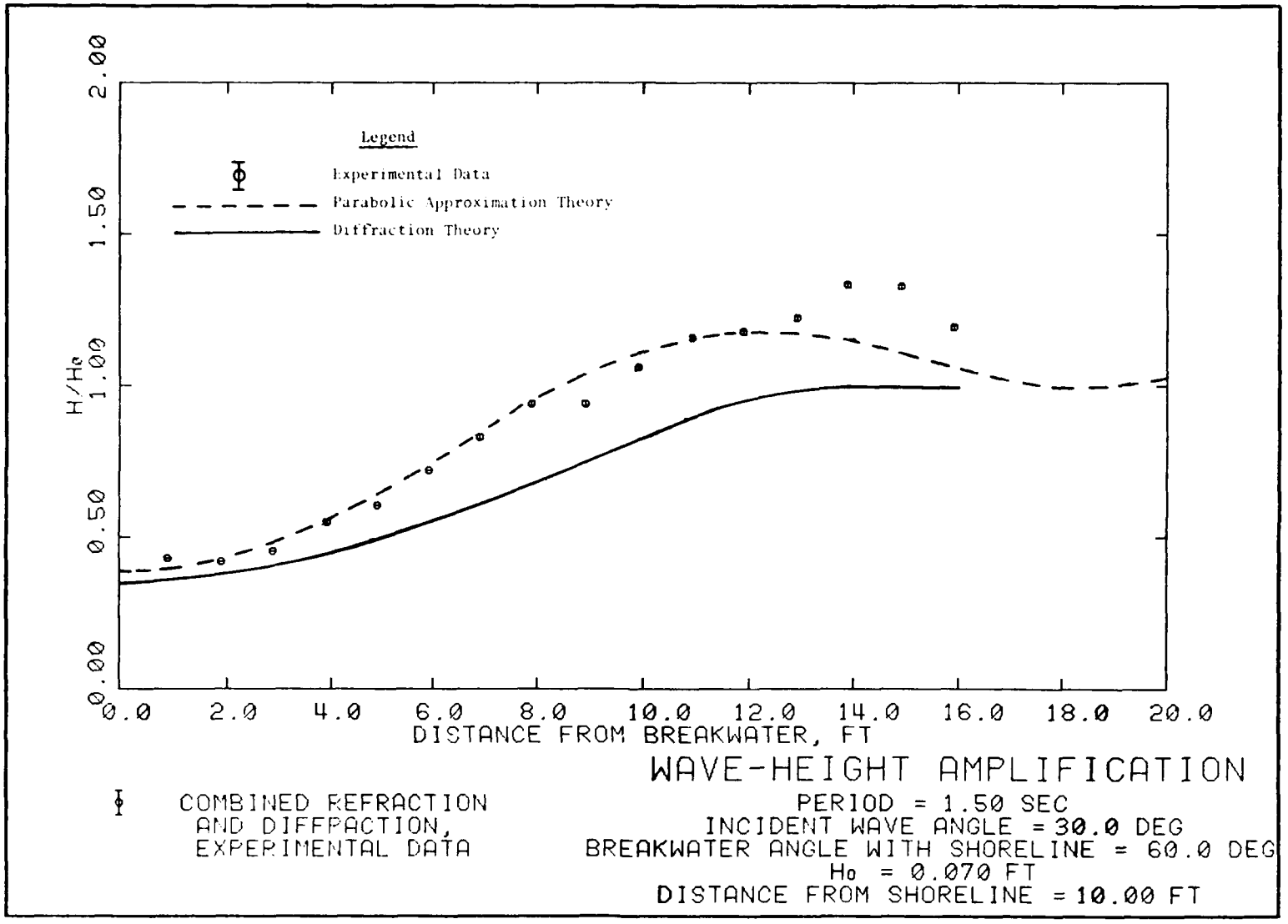
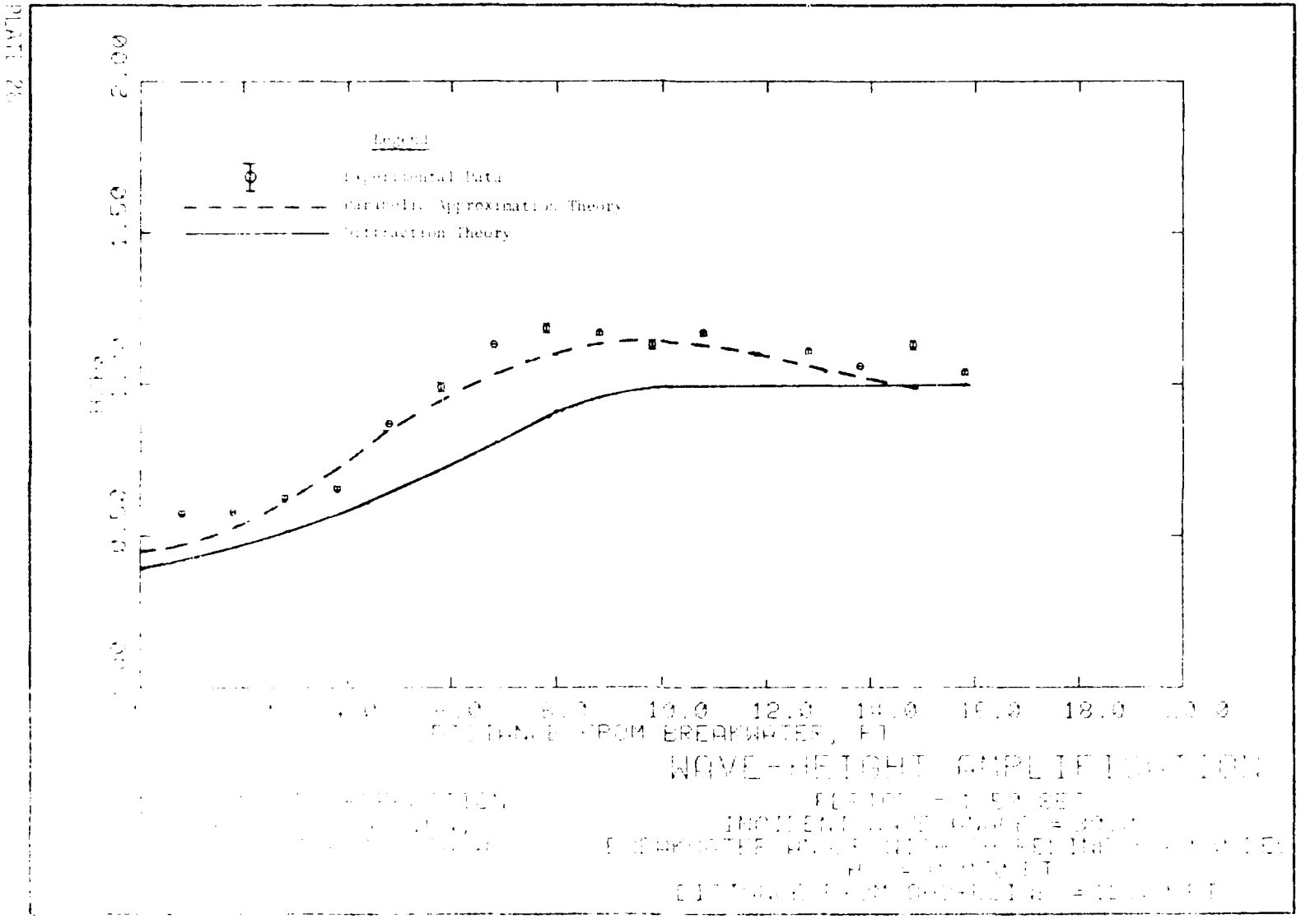
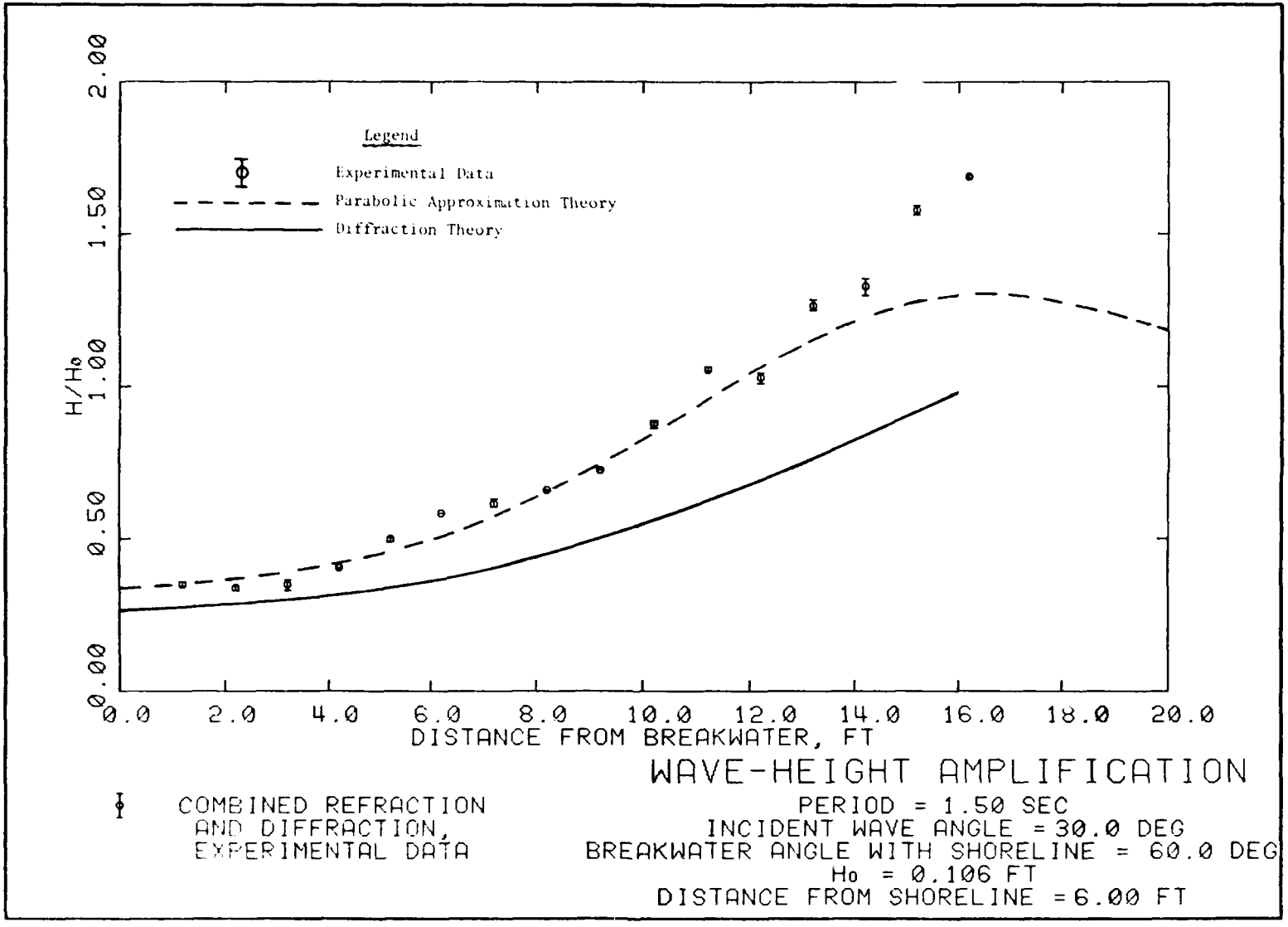

Legend
Experimental Data
Parabolic Approximation Theory
Diffraction Theory
H/Ho
2.00
1.50
1.00
0.50
0.00
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT
WAVE-HEIGHT AMPLIFICATION
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA
PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.070 FT
DISTANCE FROM SHORELINE = 10.00 FT

PLATE 27


Legend
Experimental Data
Cnoidal Approximation Theory
Refraction Theory
2.00
1.50
1.00
0.50
0.0
10.0
12.0
14.0
16.0
18.0
DISTANCE FROM BREAKWATER, FT
WAVE-HEIGHT AMPLIFICATION



WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.106 FT
DISTANCE FROM SHORELINE = 6.00 FT

PLATE 25

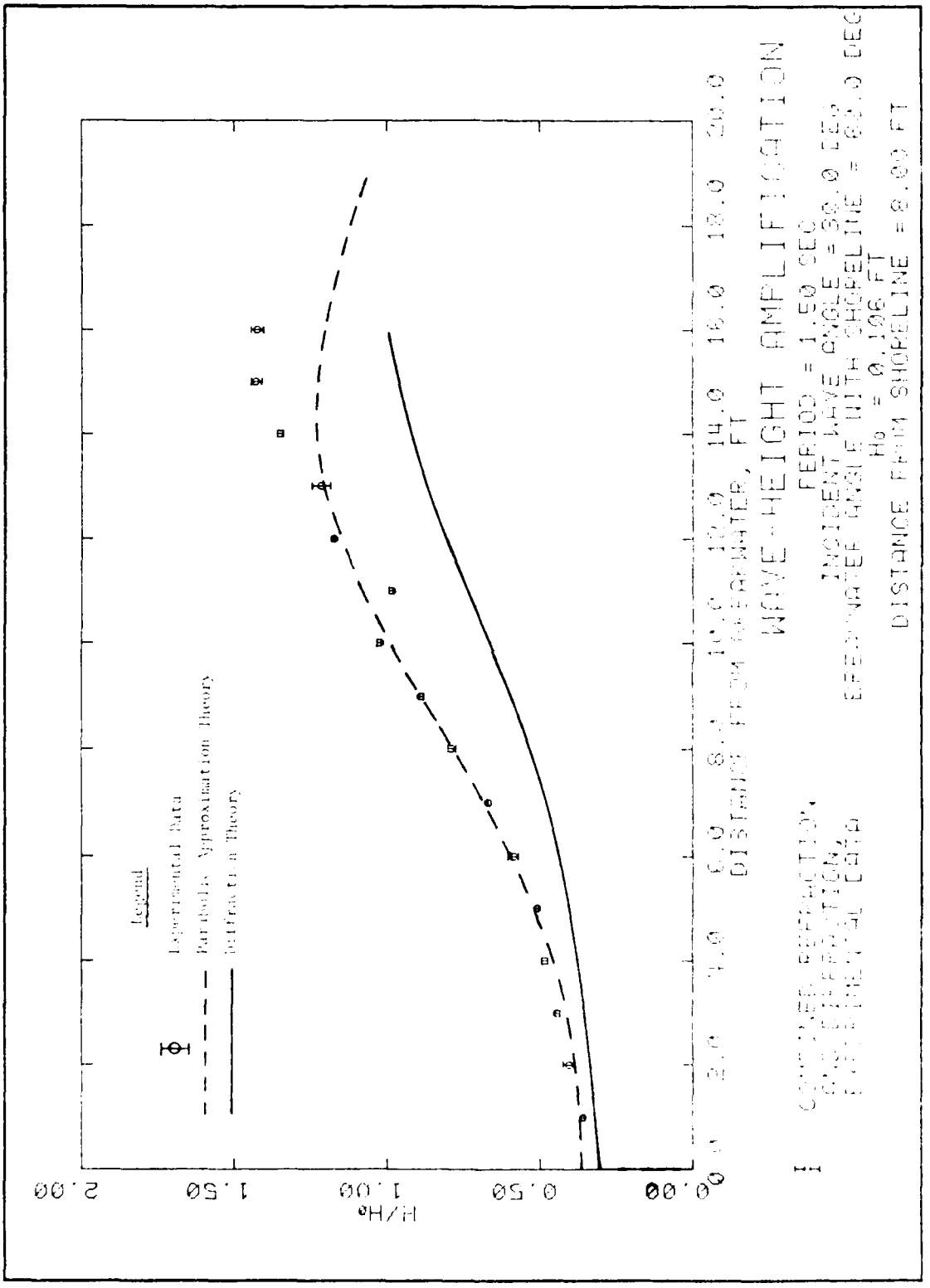
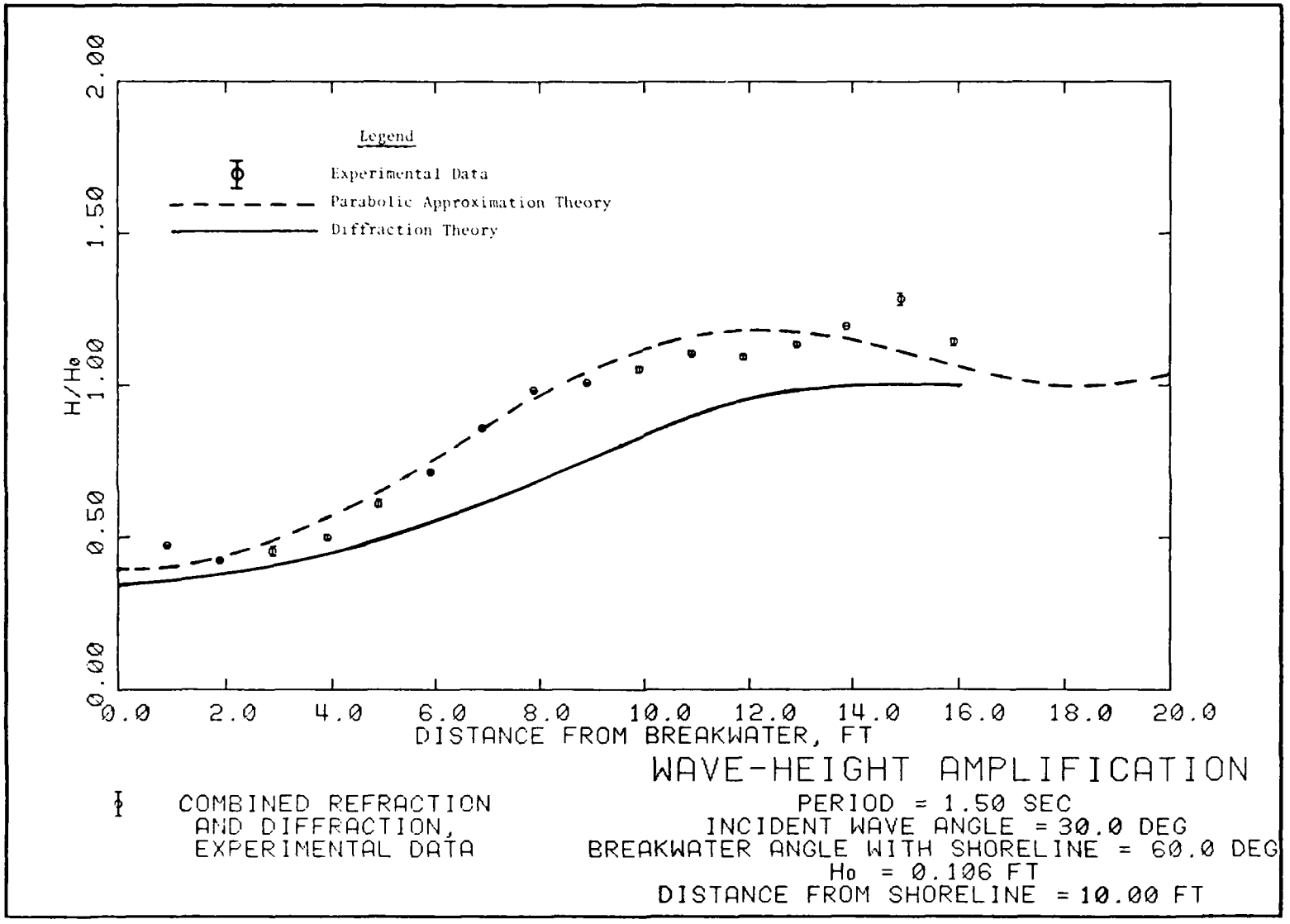

WAVE-HEIGHT AMPLIFICATION
PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER GAP WIDTH SHORELINE = 60.0 DEG
H_b = 0.106 FT
DISTANCE FROM SHORELINE = 8.00 FT
Legend
Experimental Data
Parabolic Approximation Theory
Refraction Theory
H/H_b
0.00
0.50
1.00
1.50
2.00
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0

PLATE 30



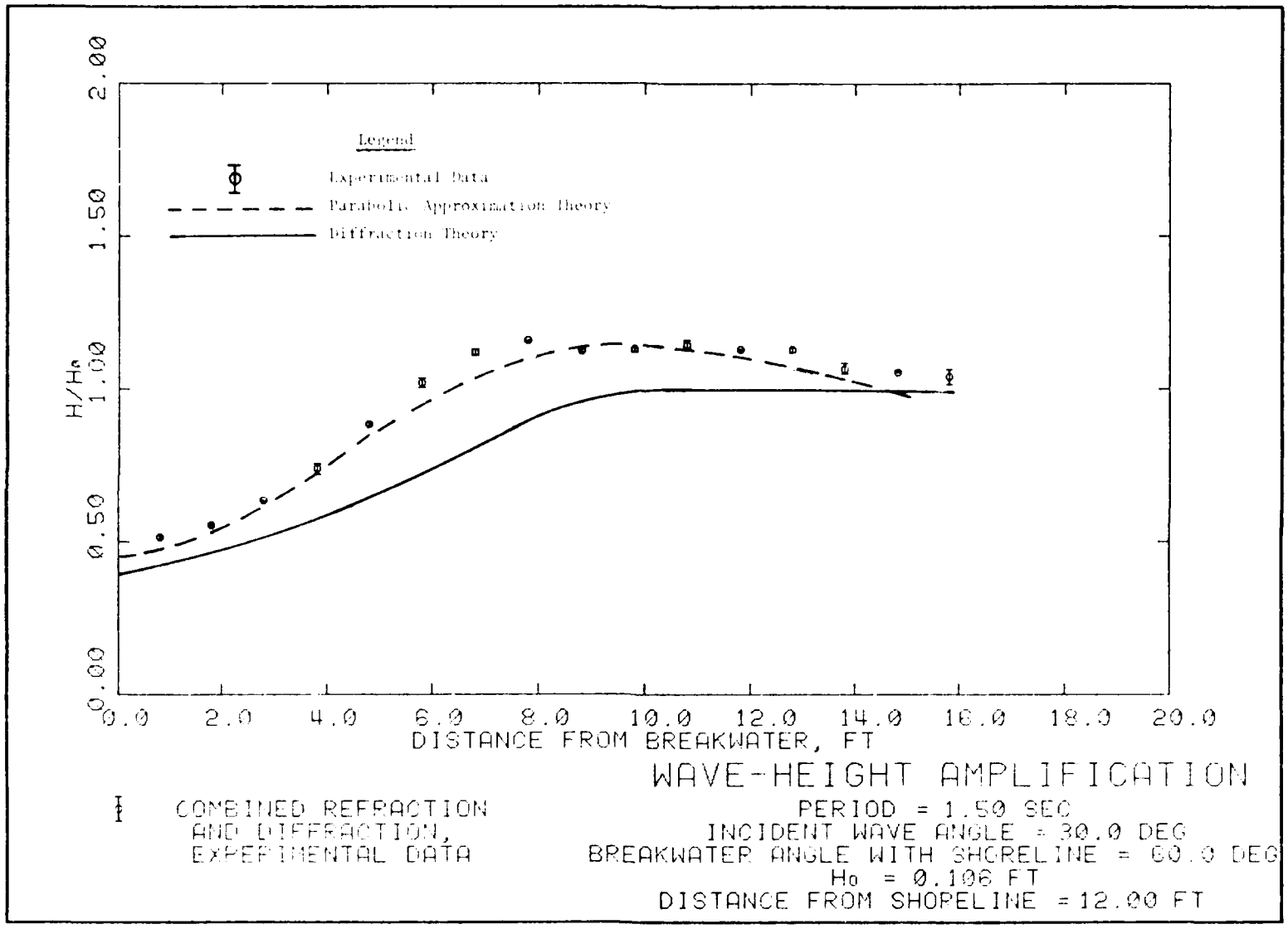
WAVE-HEIGHT AMPLIFICATION

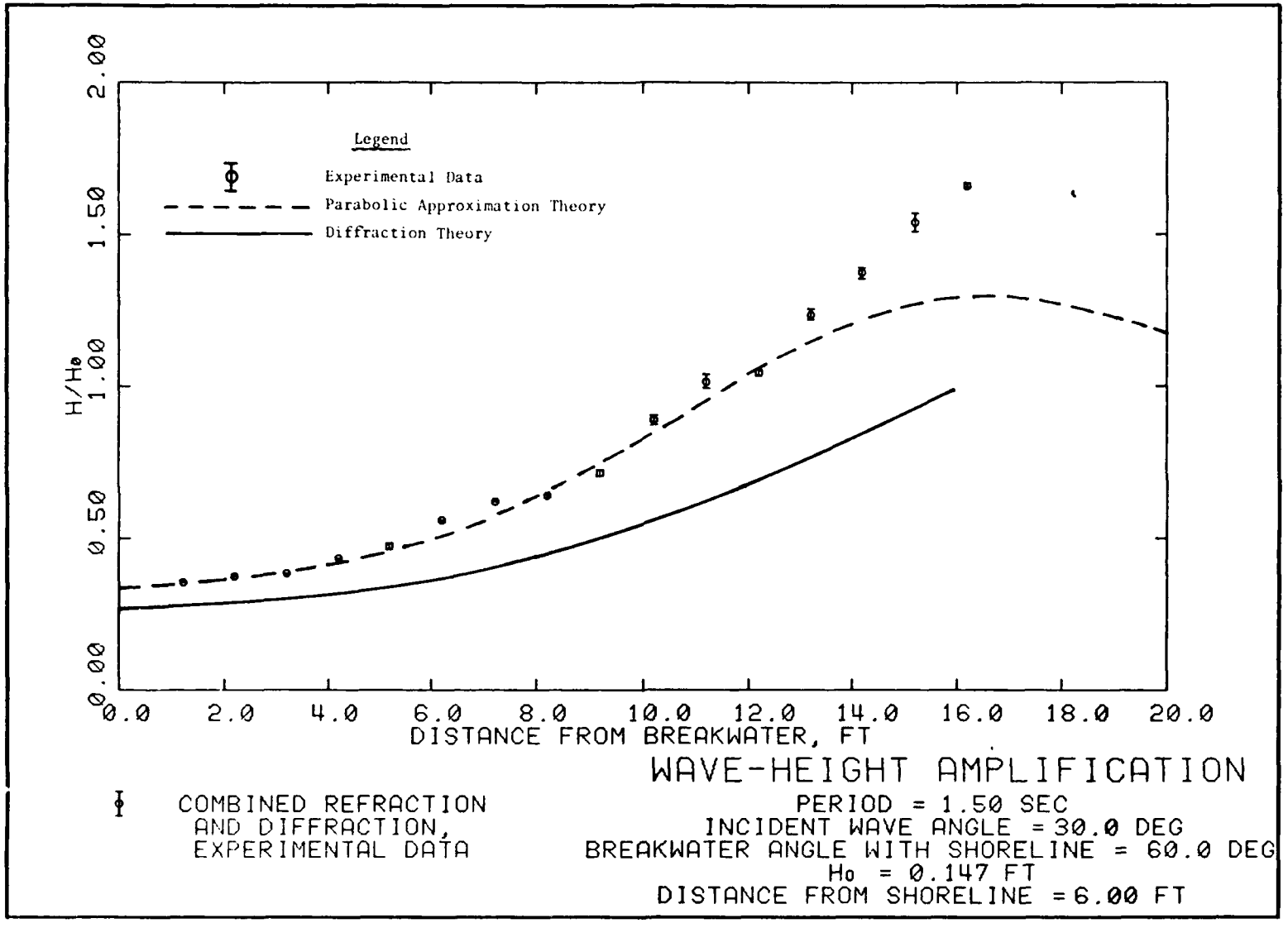
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.106 FT
DISTANCE FROM SHORELINE = 10.00 FT

PLATE 31

PLATE 32


Legend
Experimental Data
Parabolic Approximation Theory
Diffraction Theory
H/Ho
2.00
1.50
1.00
0.50
0.00
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
DISTANCE FROM BREAKWATER, FT
WAVE-HEIGHT AMPLIFICATION
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA
PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
Ho = 0.106 FT
DISTANCE FROM SHOPELINE = 12.00 FT



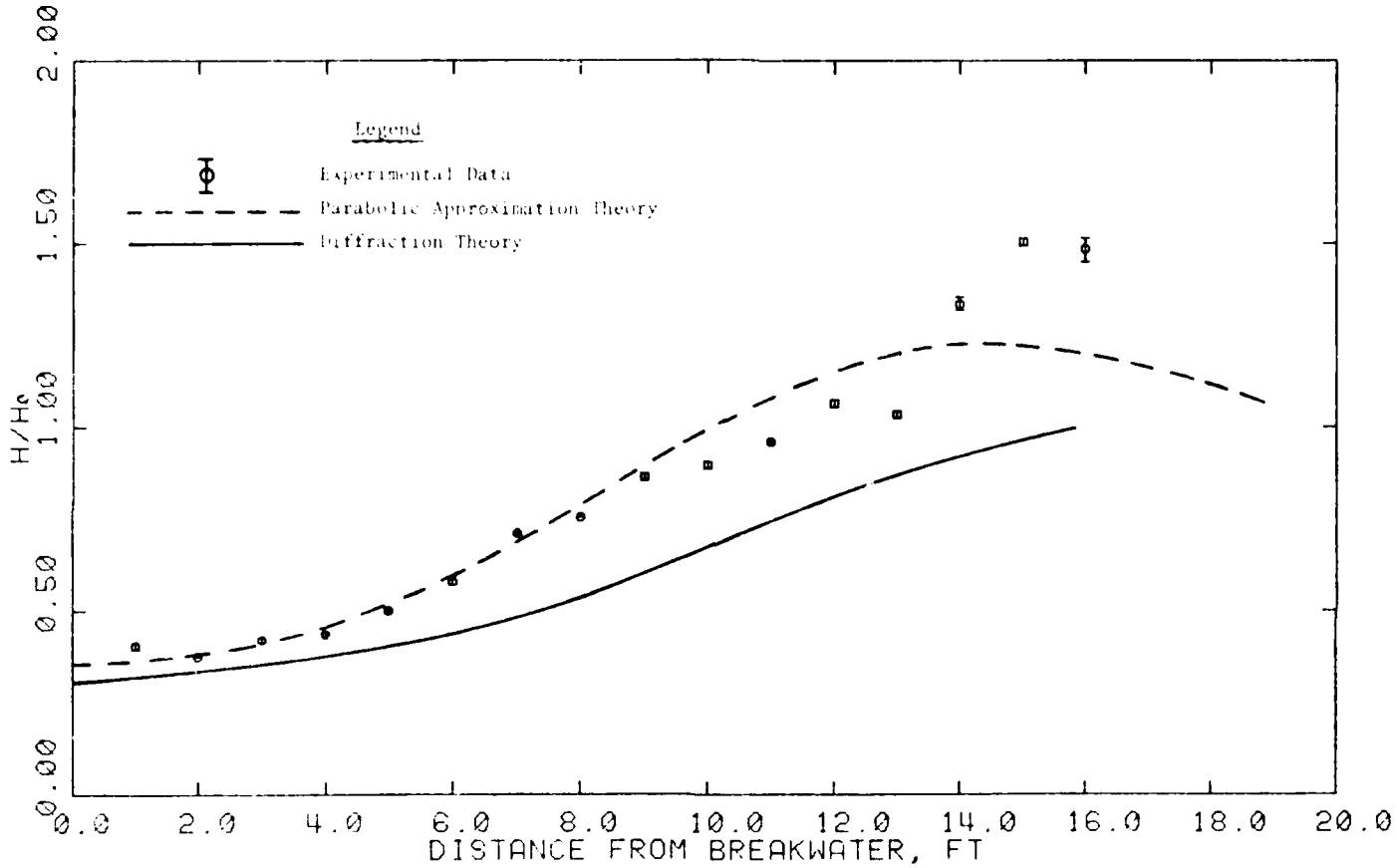
WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.147 FT
DISTANCE FROM SHORELINE = 6.00 FT

PLATE 33

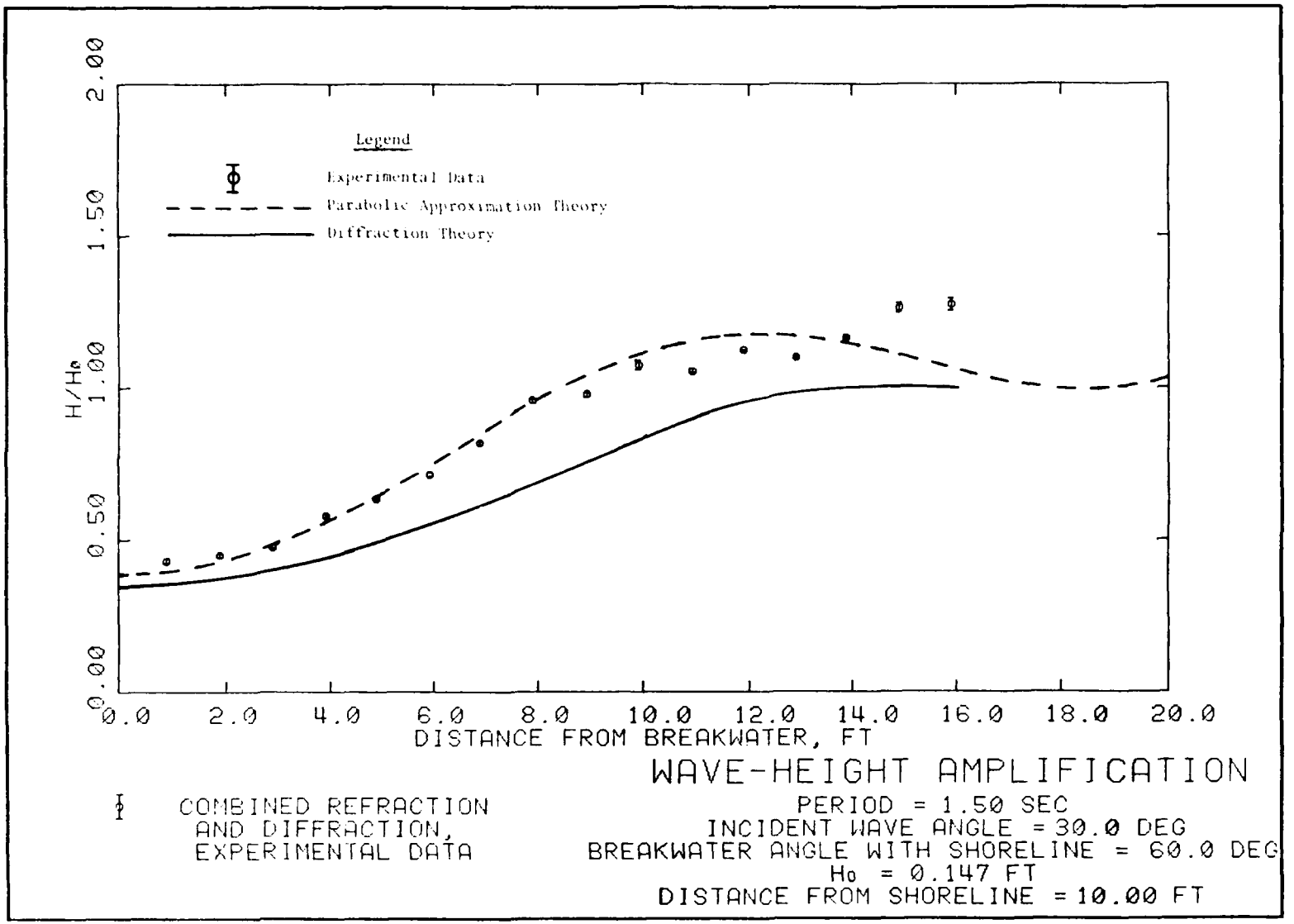
PLATE 34



WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.147 FT
DISTANCE FROM SHORELINE = 8.00 FT



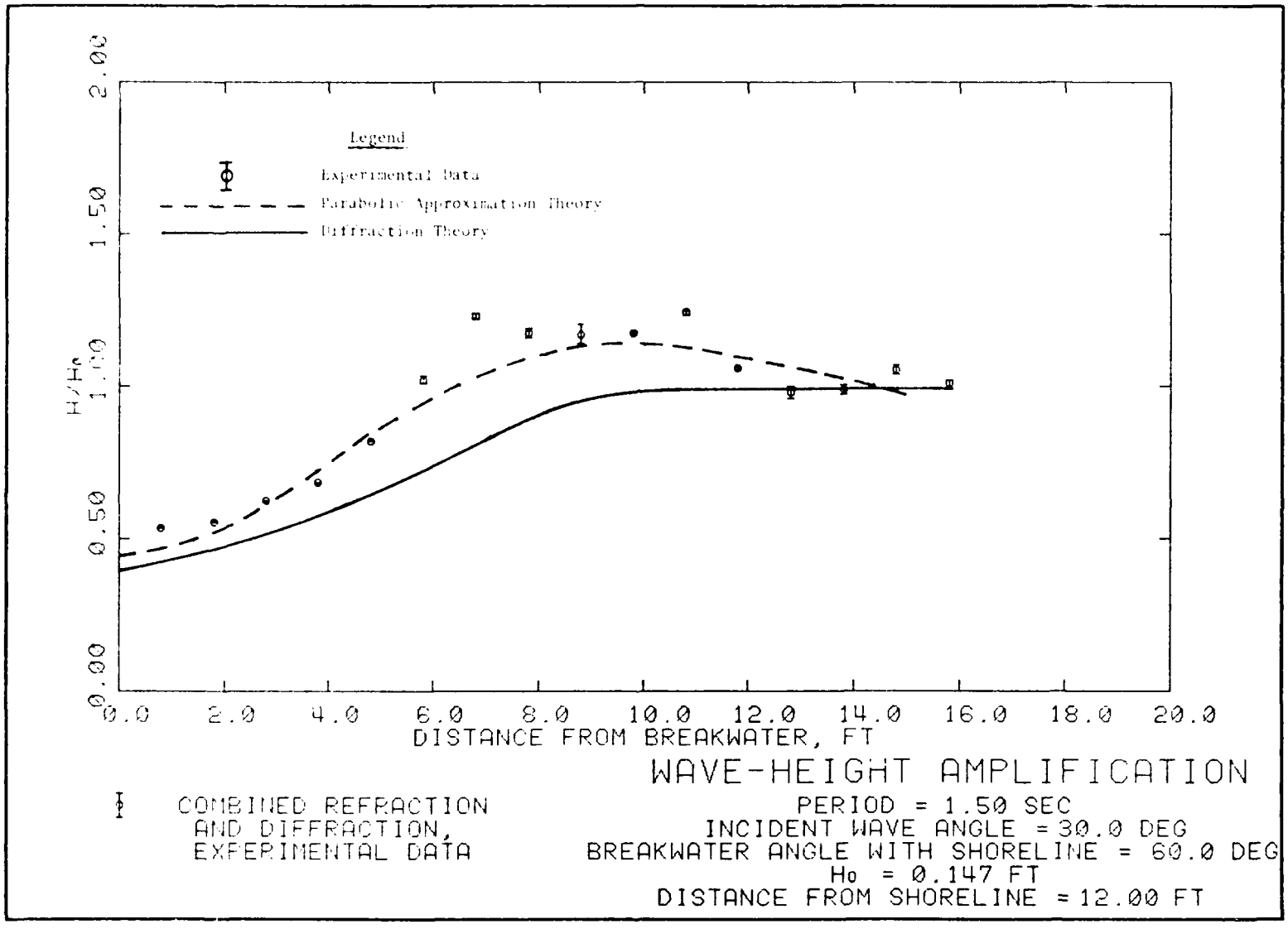
WAVE-HEIGHT AMPLIFICATION

COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_o = 0.147 FT
DISTANCE FROM SHORELINE = 10.00 FT

PLATE 35

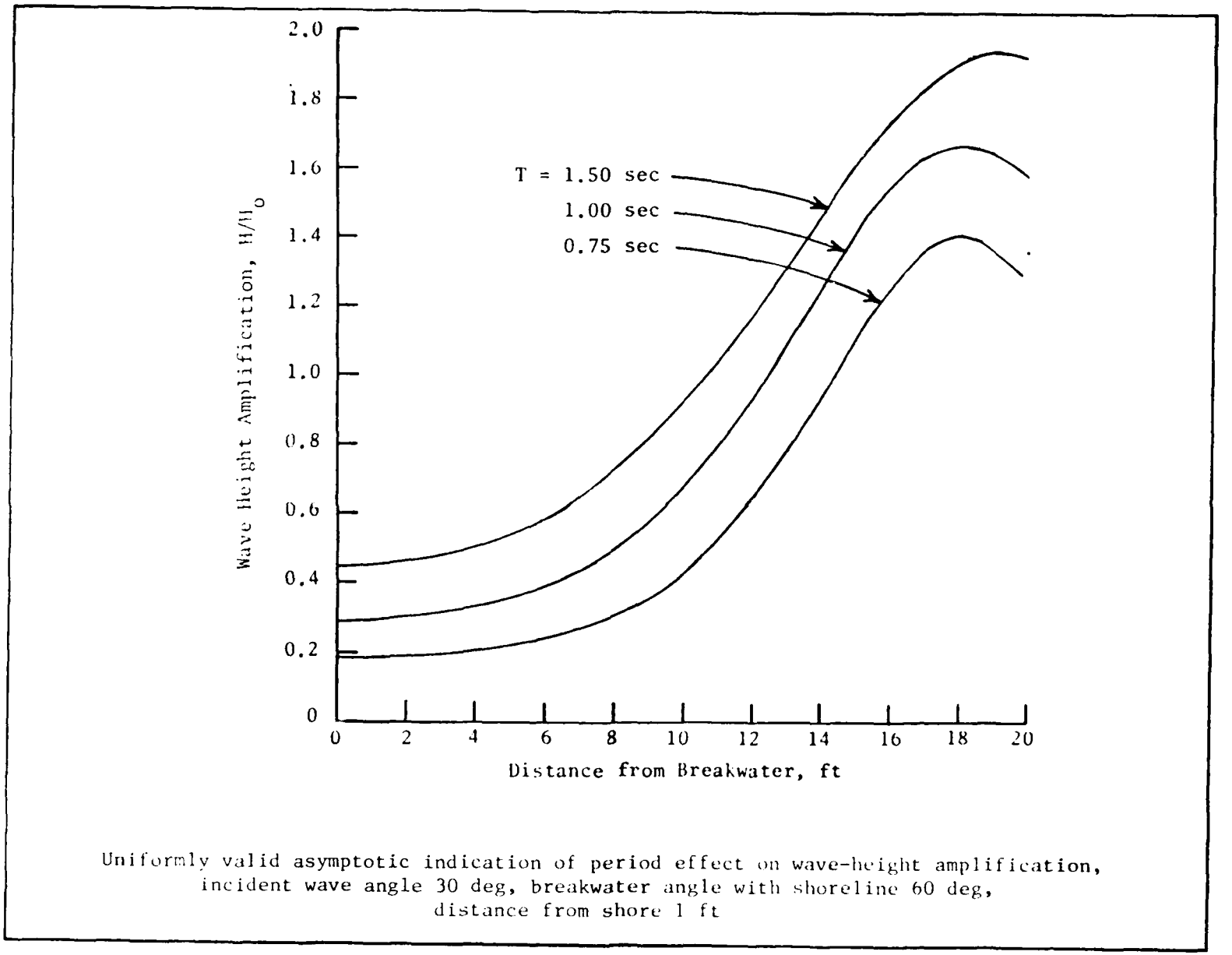
PLATE 36



WAVE-HEIGHT AMPLIFICATION

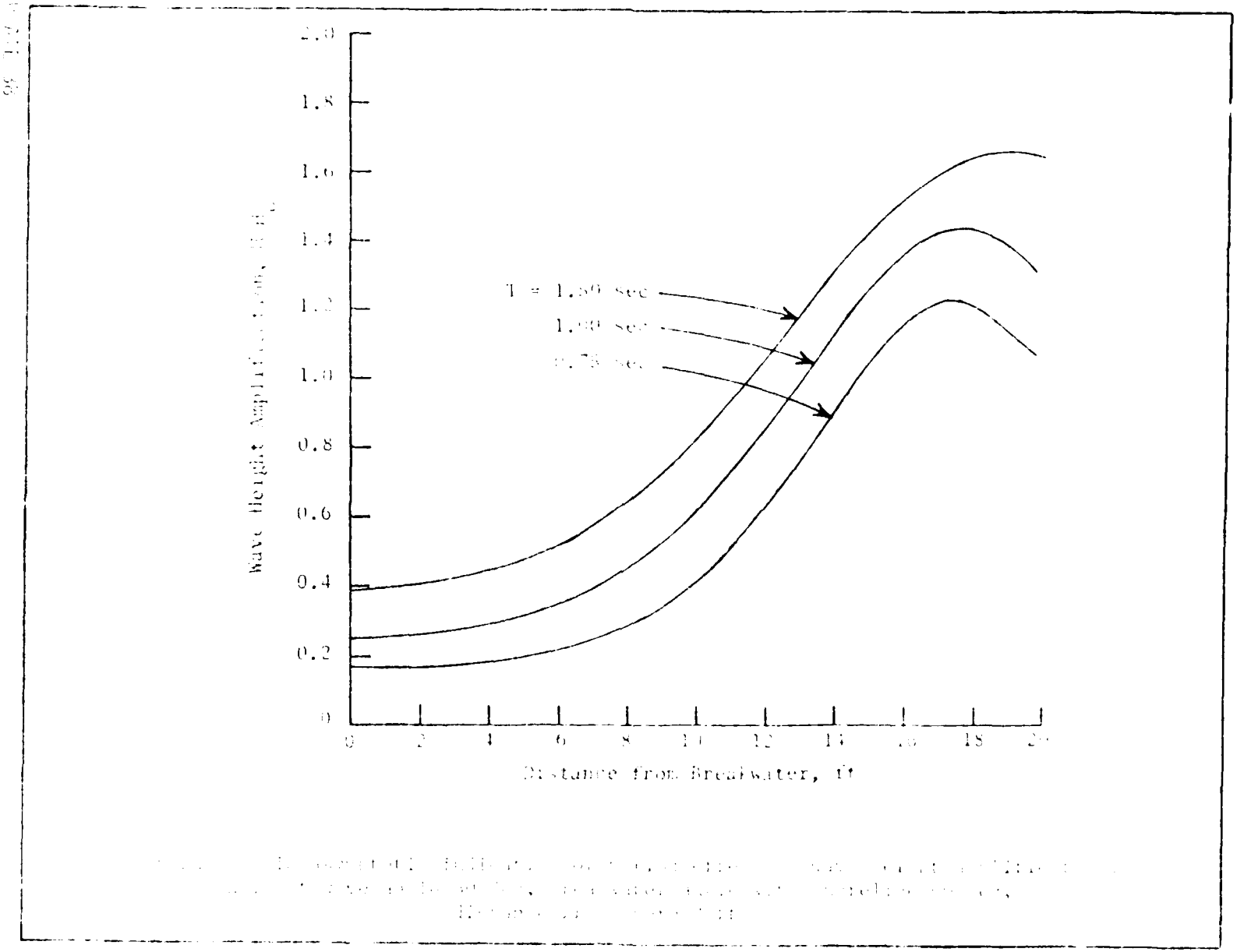
COMBINED REFRACTION AND DIFFRACTION, EXPERIMENTAL DATA

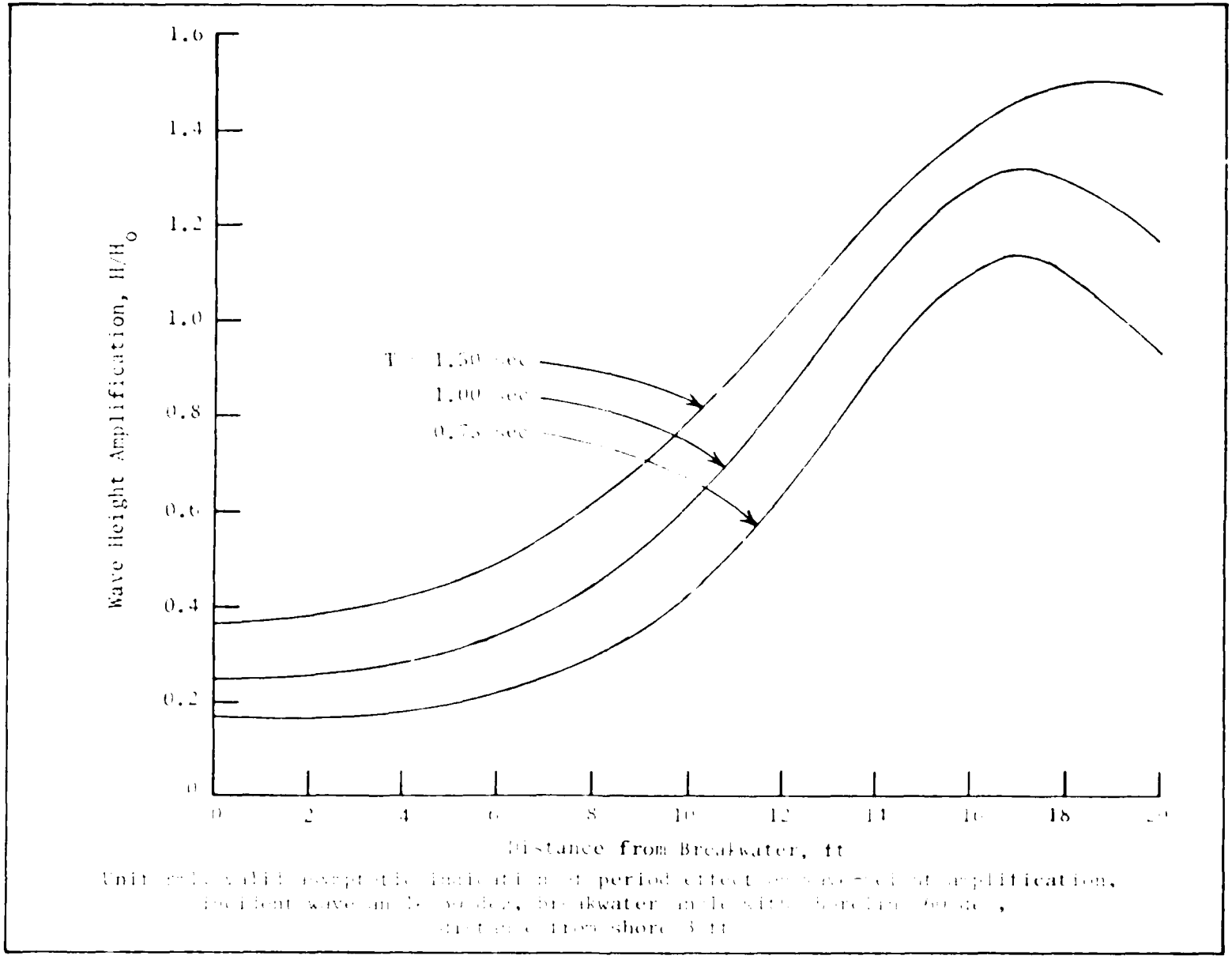
PERIOD = 1.50 SEC
INCIDENT WAVE ANGLE = 30.0 DEG
BREAKWATER ANGLE WITH SHORELINE = 60.0 DEG
H_0 = 0.147 FT
DISTANCE FROM SHORELINE = 12.00 FT



Uniformly valid asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 60 deg, distance from shore 1 ft

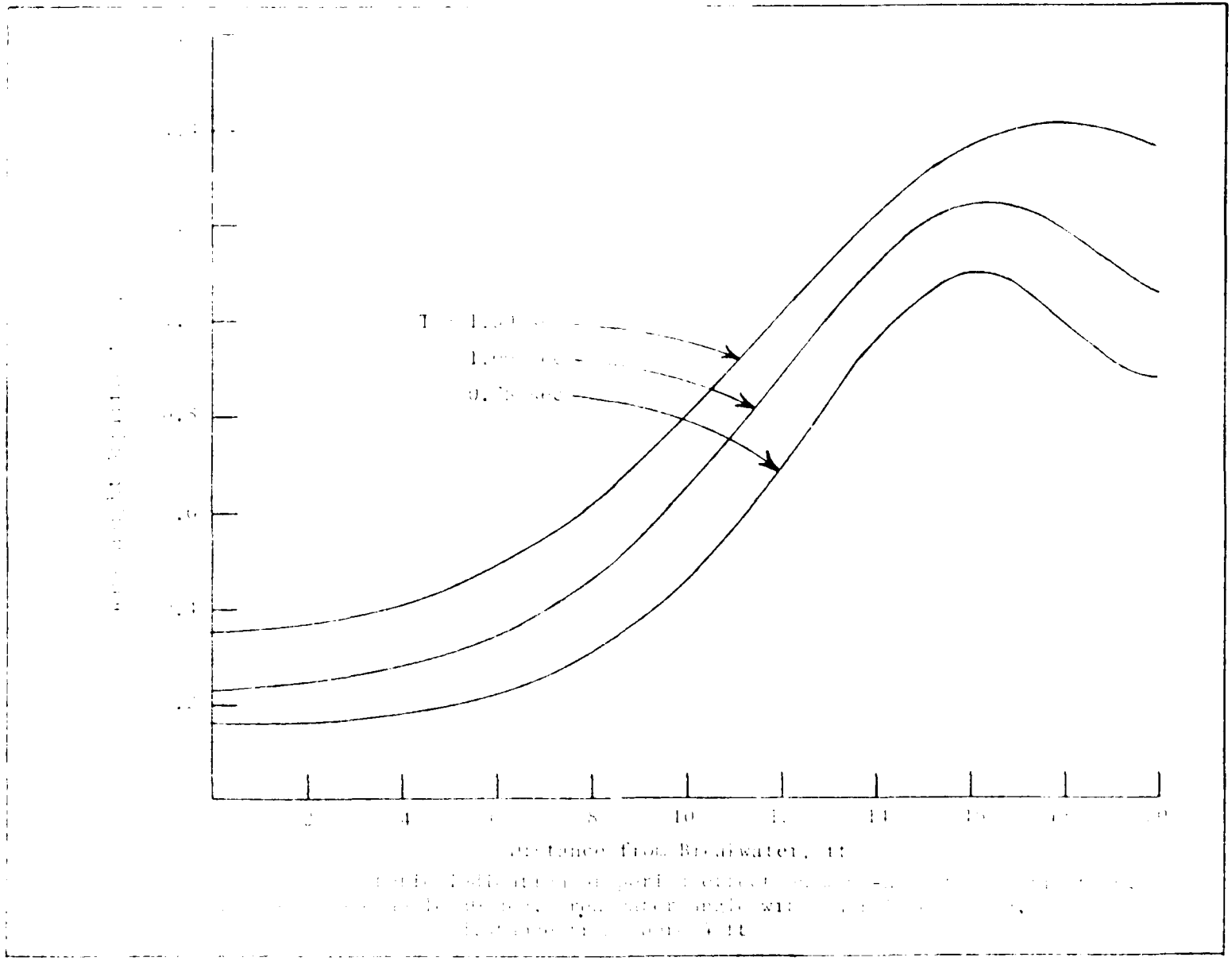
FLATL 3/

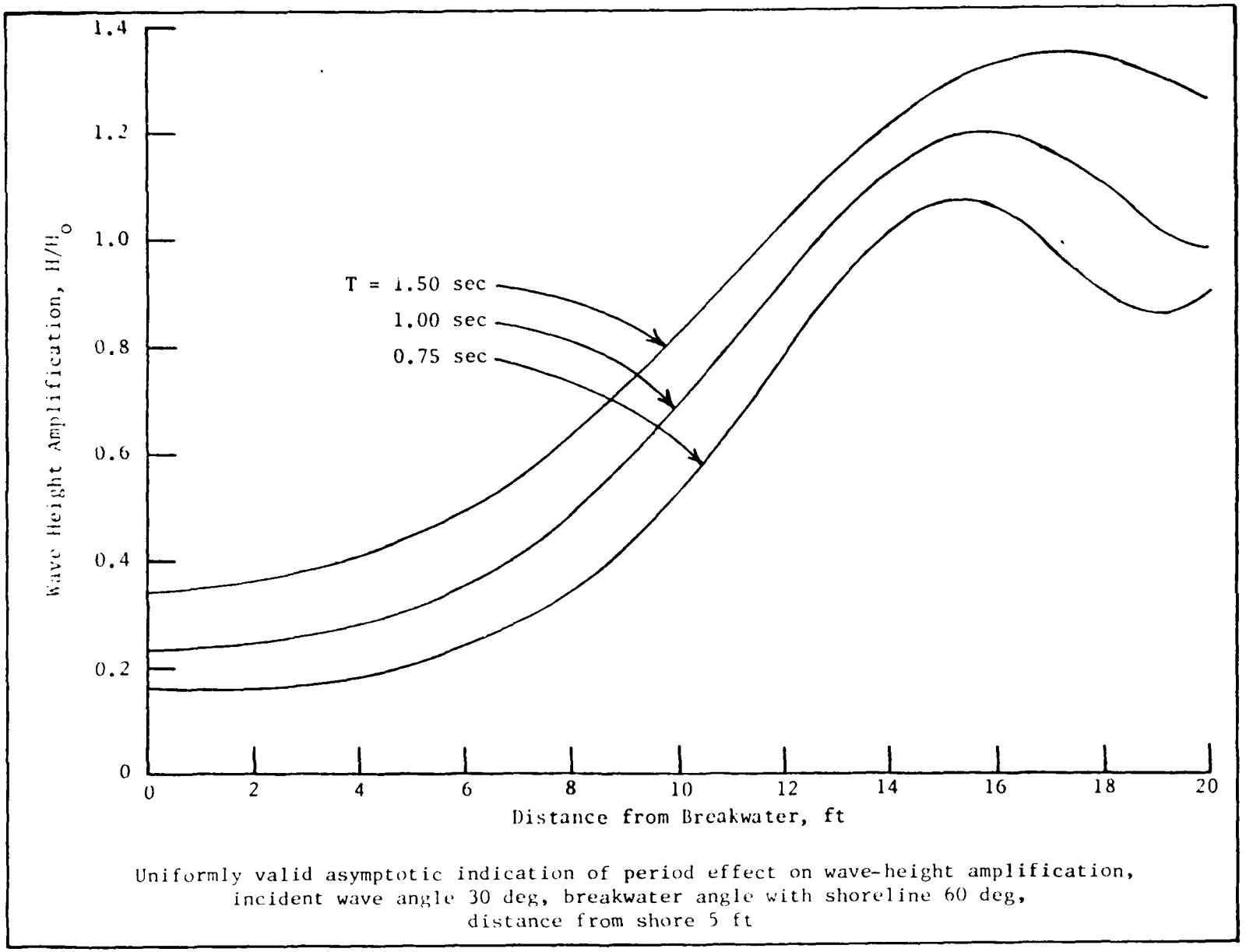

2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
2
4
6
8
12
18
T = 1.50 sec
1.30 sec
Distance from Breakwater, ft



Unit cell solid representing fundamental period effect on wave height amplification, incident wave angle 0 degrees, breakwater gap width 6 feet, distance from shore 3 ft

PLATE 39

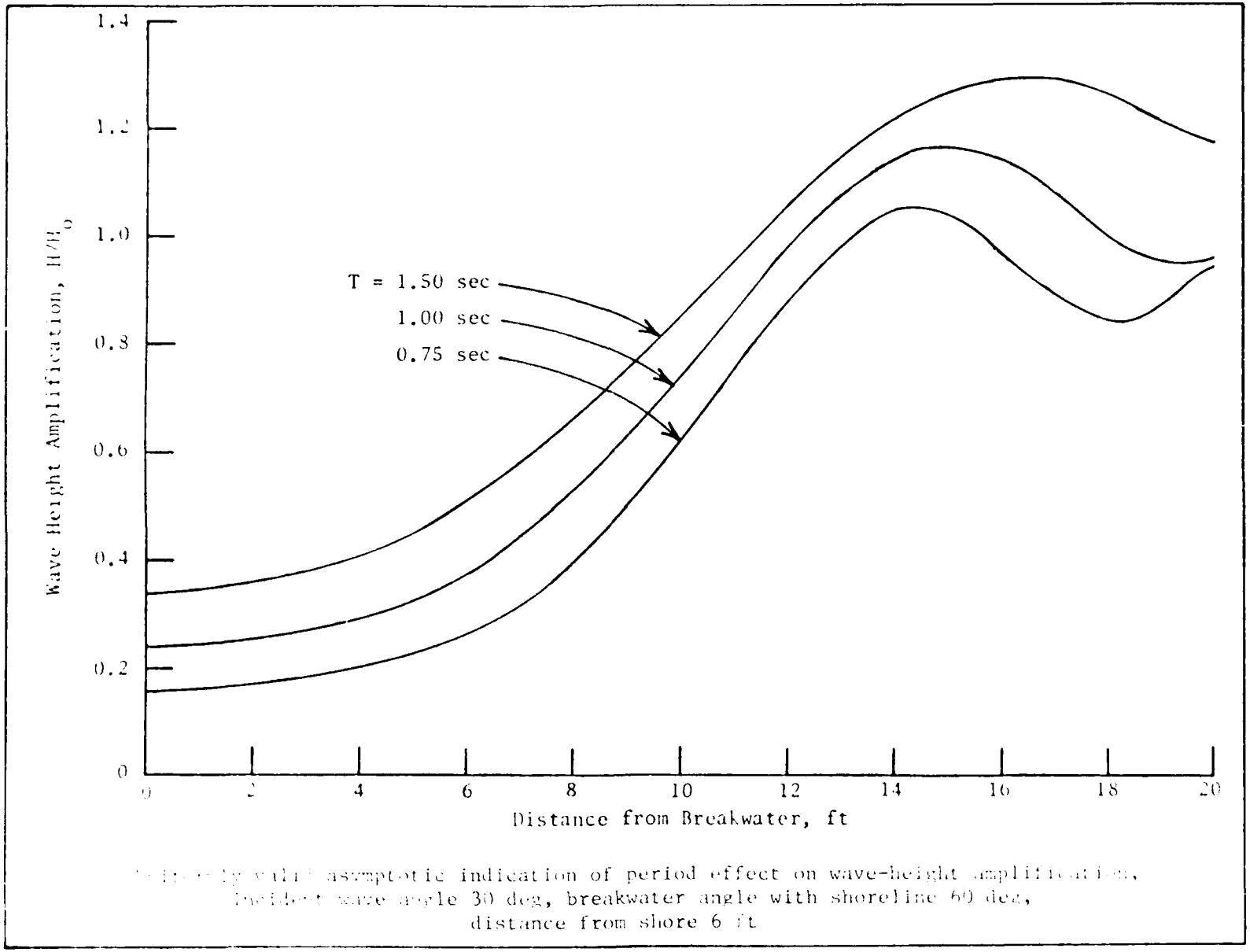




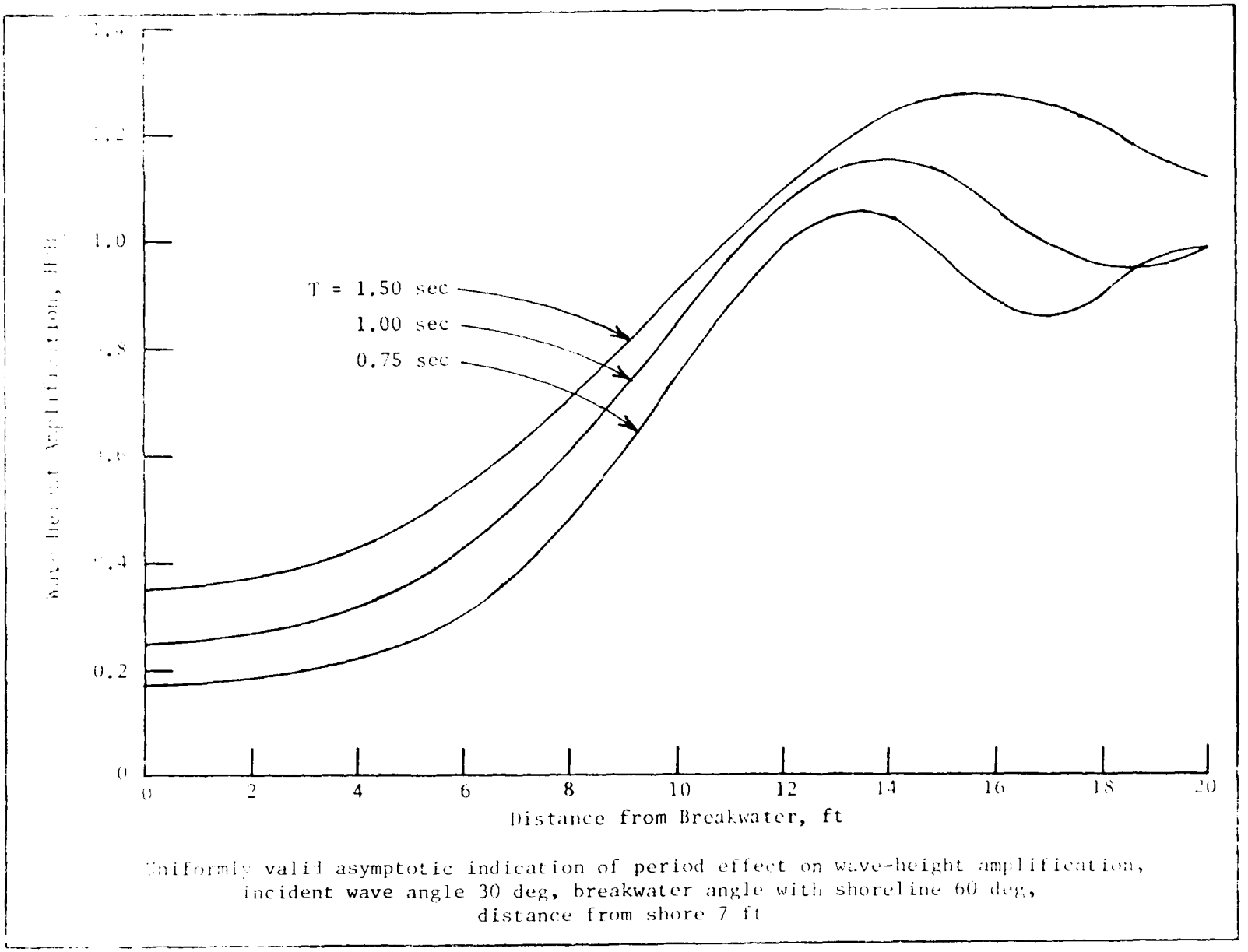
Uniformly valid asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 60 deg, distance from shore 5 ft

PLATE 41

PLATE 42

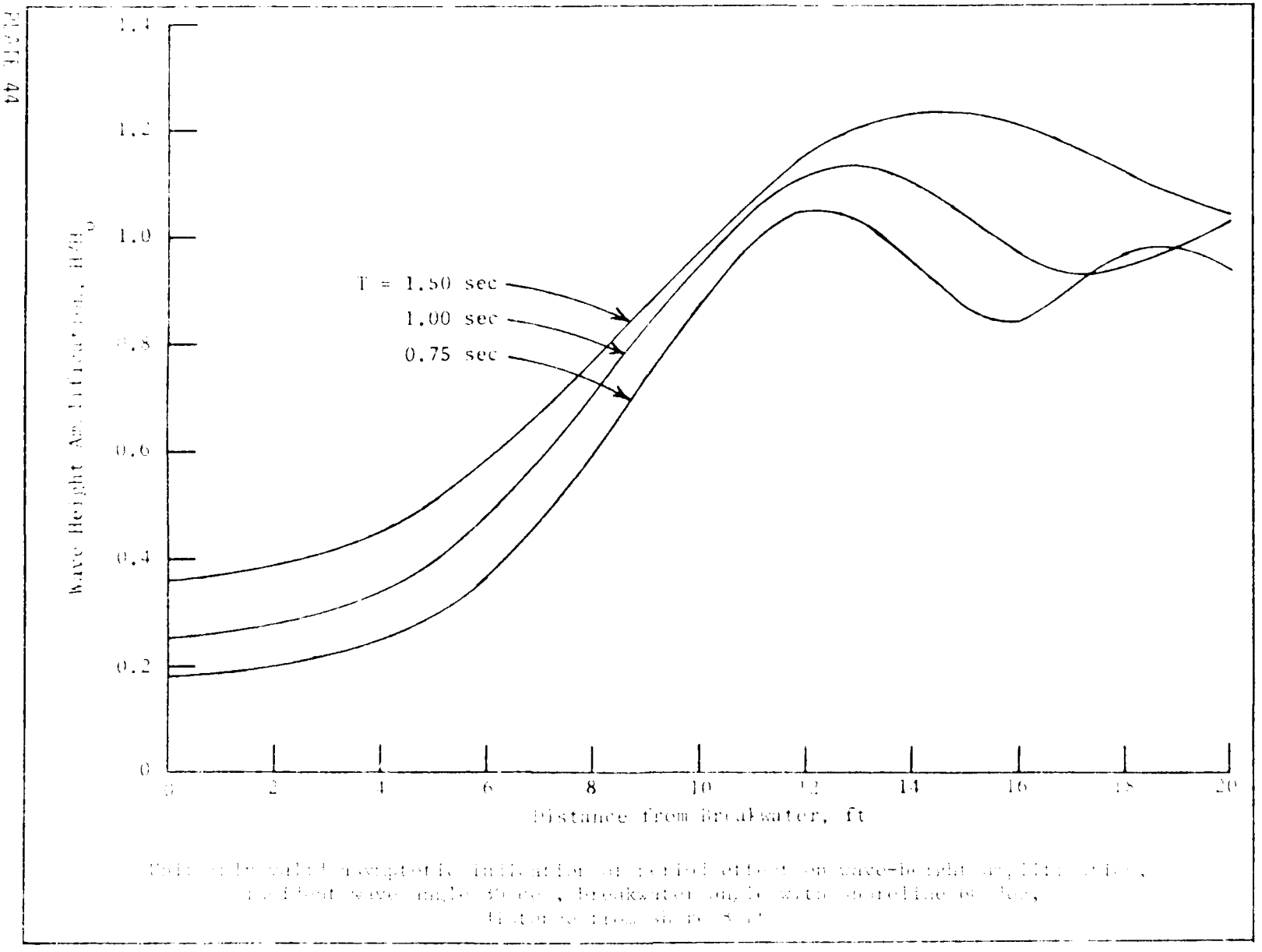


[illegible] with asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 60 deg, distance from shore 6 ft

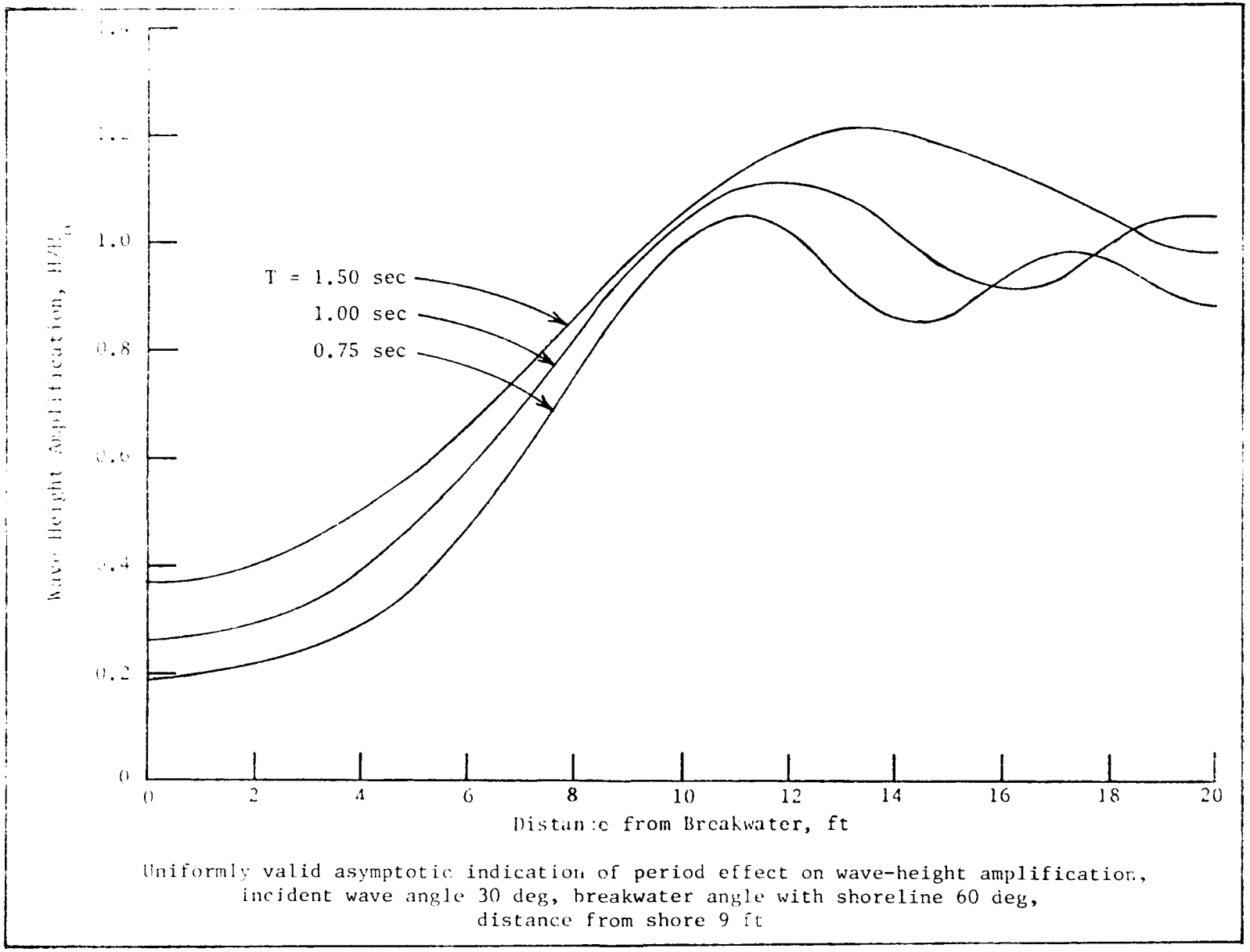


Uniformly valid asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 60 deg, distance from shore 7 ft

PLATE 33



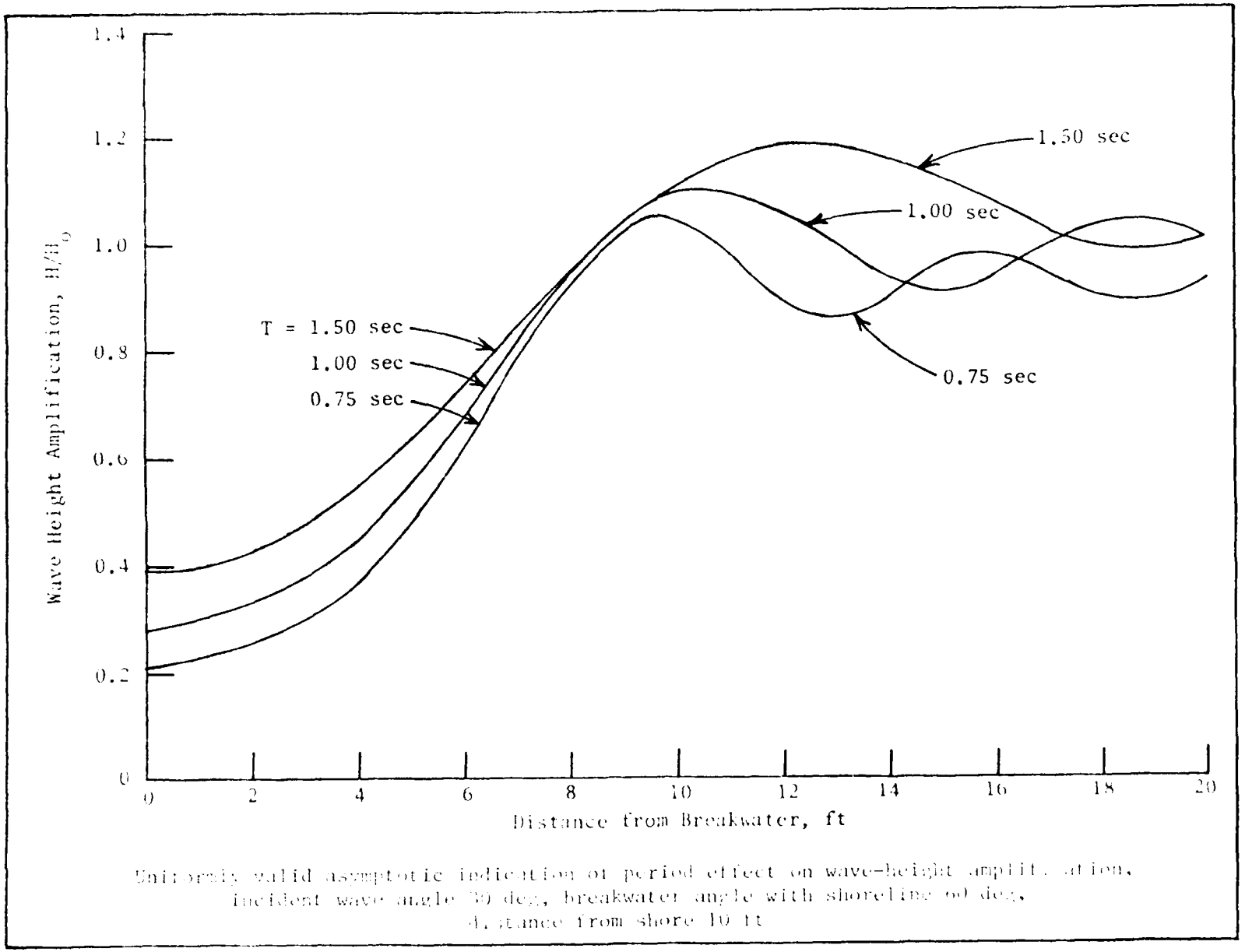
Data on Periodic Synoptic Variation of Period Effect on Wave-Height Amplification,
1:1 Plane Wave Angle Plane, Breakwater Angle with Coastline of 90°,
Distance from Shore 5.0'



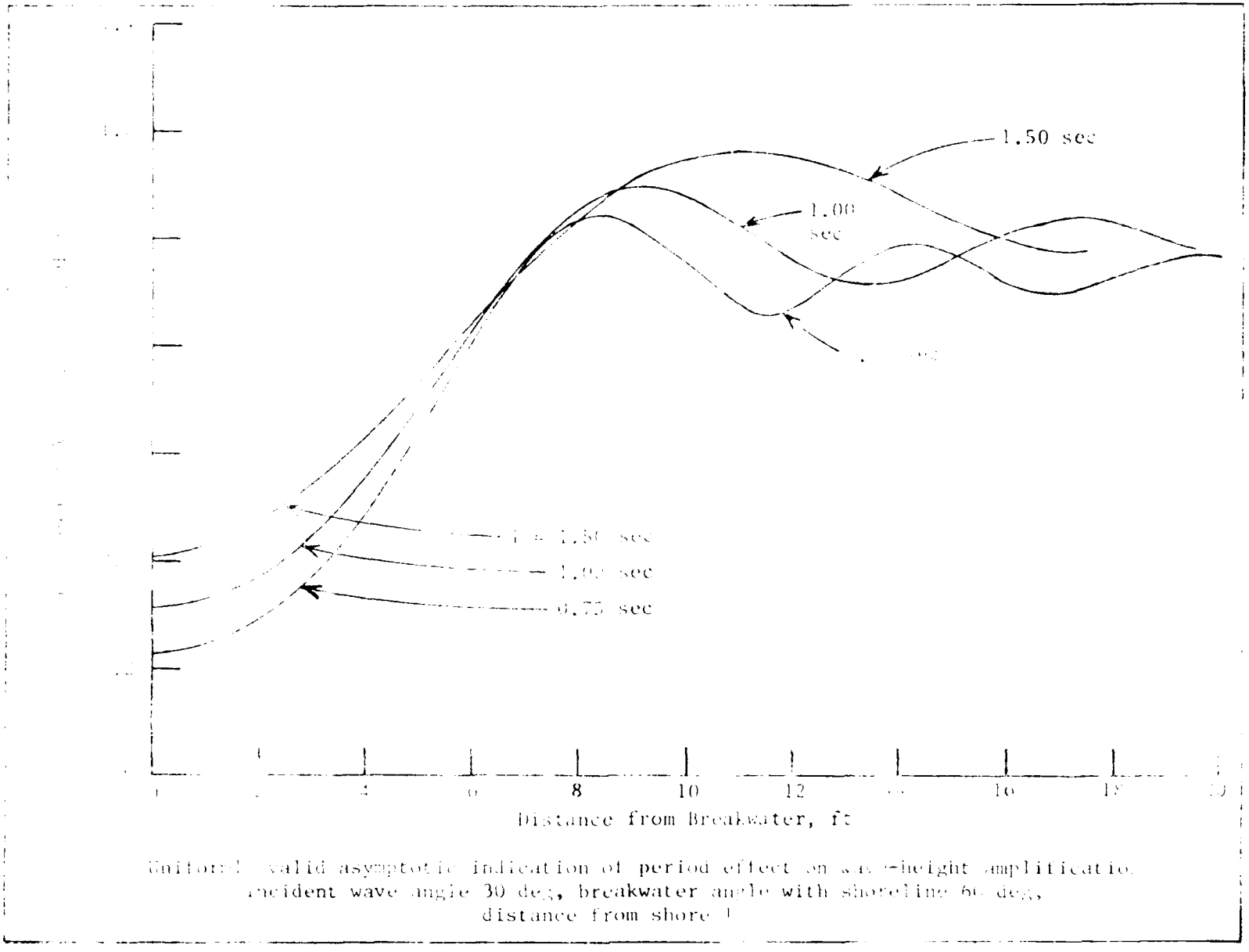
Uniformly valid asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 60 deg, distance from shore 9 ft

PLATE 45

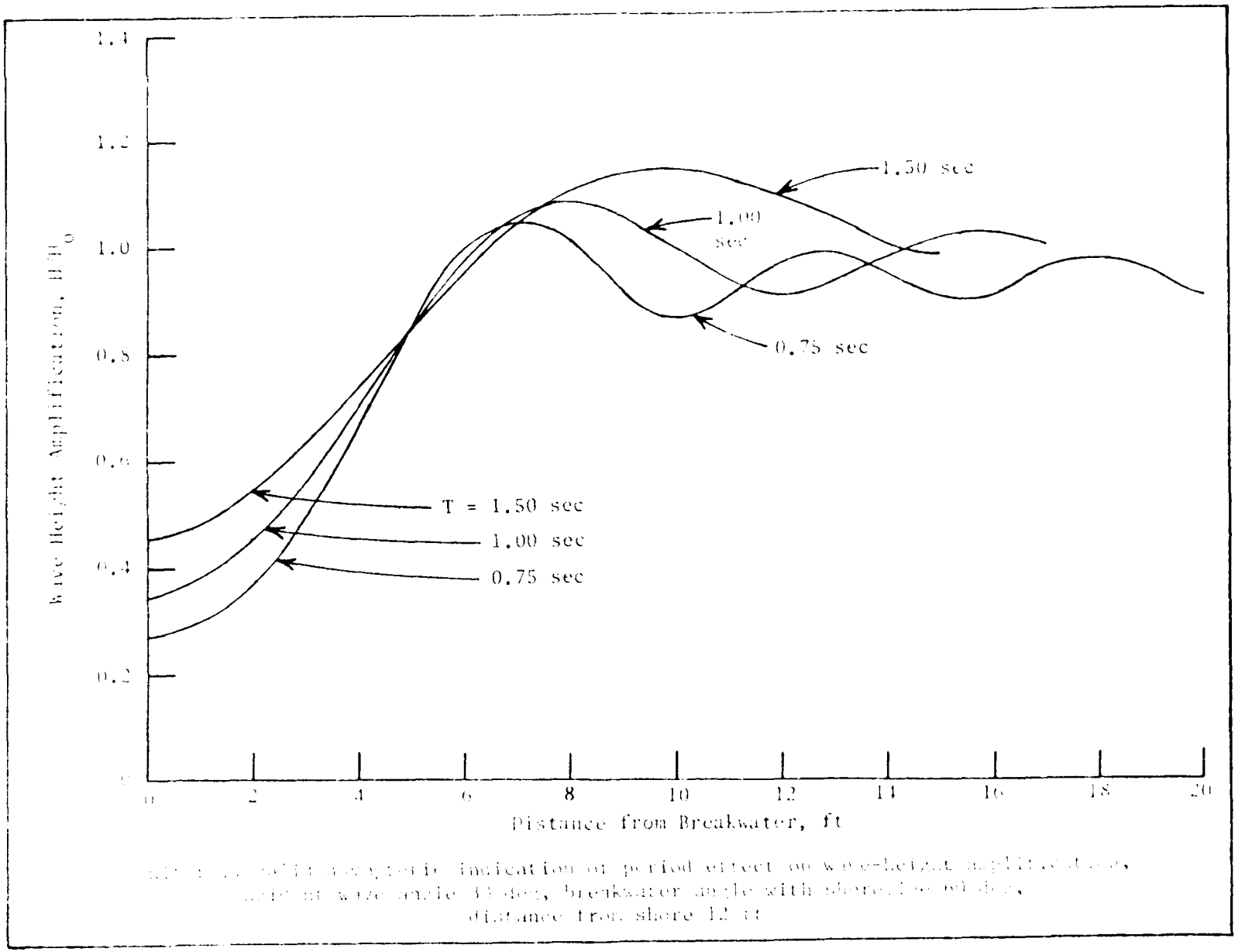
PLATE 46



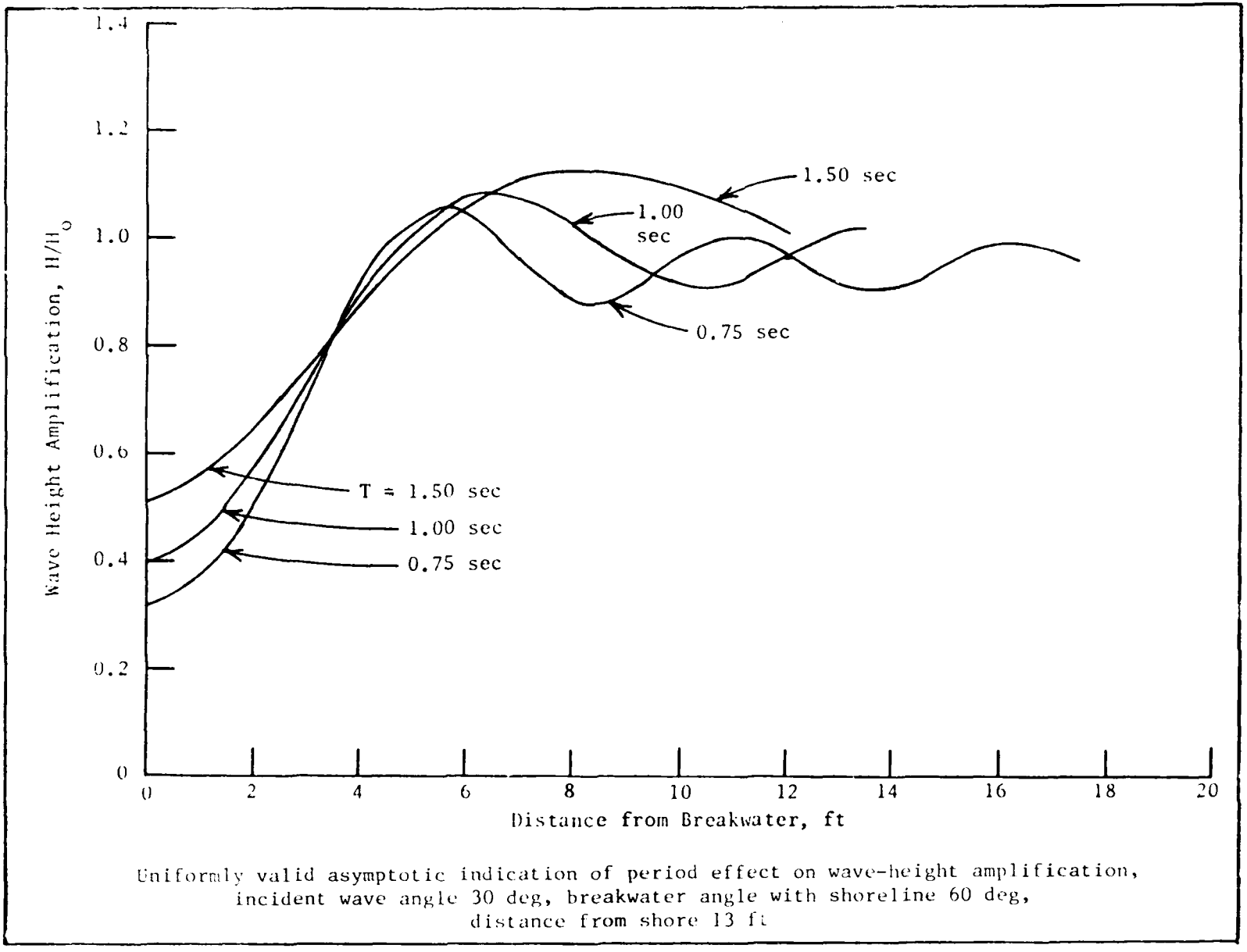
Universally valid asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 60 deg, distance from shore 10 ft



Uniform valid asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 60 deg, distance from shore



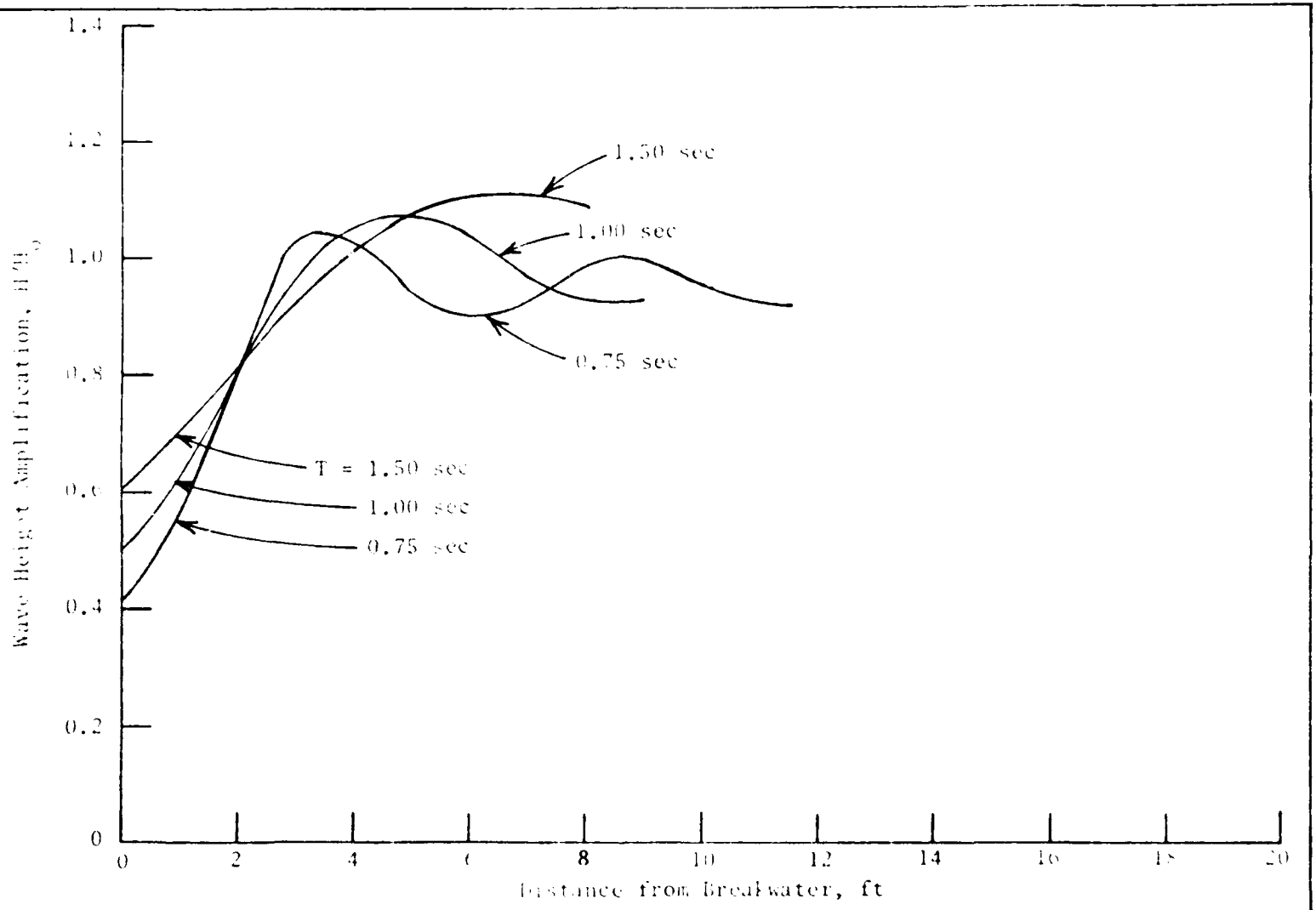
Typical experimental indication of period effect on wave-height amplification, angle of wave crest 33 deg, breakwater angle with shoreline 90 deg, distance from shore 12 ft



Uniformly valid asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 60 deg, distance from shore 13 ft

PLATE 49

PLATE 50



Uniformly valid asymptotic indication of period effect on wave-height amplification, incident wave angle 30 deg, breakwater angle with shoreline 90 deg, distance from shore 14 ft

APPENDIX A: NOTATION

$\bar{K}_t$	Arbitrary radiated wave parameter for straight breakwater, 1/ft
L	Wavelength at arbitrary location, ft
L_o	Incident wavelength, ft
L_r	Wavelength reflected from breakwater, ft
L_t	Wavelength radiated from breakwater tip, ft
n	Ratio of group velocity, c_g, to wave celerity, c, dimensionless
P	Rate of wave power transmission, lb/sec
r	Radial distance, ft
$\bar{r}$	Phase function parameter, ft
R	Phase function of waves radiated from curved breakwater tip, deg
R_s	Phase function of waves radiated from straight breakwater tip, deg
s	Bottom slope, dimensionless
S	Phase function of incident wave for curved breakwater, deg
S_s	Phase function of incident wave for straight breakwater, deg
$S_2(\rho_1^2)$	Fresnel sine integral, dimensionless
$\bar{S}$	Phase function of wave reflected from curved breakwater, deg
$\bar{S}_s$	Phase function of wave reflected from straight breakwater, deg
t	Time element, sec
T	Wave period, sec
u_1	Diffraction integration limits, dimensionless
u_2	Diffraction integration limits, dimensionless
$\bar{u}$	Two-dimensional horizontal velocity vector, ft/sec
x	Horizontal direction in Cartesian coordinate system, ft
y	Horizontal direction in Cartesian coordinate system, ft
z	Vertical direction in Cartesian coordinate system, ft
y	Arbitrary parameter, dimensionless
δ	Battjes wave number correction factor $(a_{xx} + a_{yy})/k^2a$, dimensionless
η	Local water-surface elevation and complex wave amplitude, ft
θ	Arbitrary wave angle, deg
θ_o	Incident wave angle, deg
θ_r	Reflected wave angle along breakwater, deg
θ_t	Radiated wave angle from oscillatory point source at breakwater tip, deg
π	3.141592654, dimensionless

ρ Fluid density, [illegible]

ρ_1 Pressure integral limit

σ Pressure integral parameter, dimensionless

τ Pressure integral parameter, dimensionless

ϕ Velocity potential, [illegible]

ω Angular frequency, [illegible]

Δb Wave tank spacing, [illegible]

θ Argument of trigonometric function $\sin\theta$, [illegible]

$\bar{\theta}$ Argument of mathematical function $\sin\bar{\theta}_1$, [illegible]

∇ Horizontal gradient operator, dimensionless

∞ Infinity, dimensionless

< Less than, dimensionless

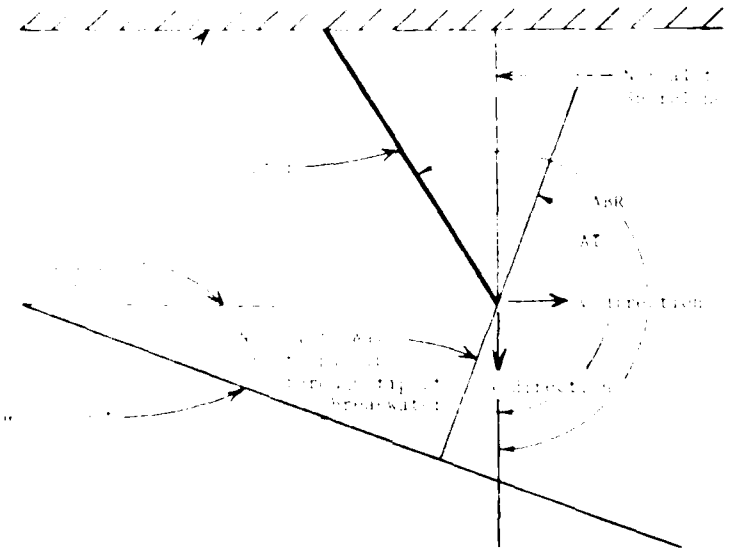
> Greater than, dimensionless

∂ Partial derivative symbol, dimensionless

[illegible] [illegible]

APPENDIX B: UNIFORM ASYMPTOTIC THEORY LIU, LOZANO, AND PANTAZARAS (1979) FOR COMBINED REFRACTION AND DIFFRACTION DOWNCOAST OF AN OBLIQUE BREAKWATER

(Program obtained by contract with Philip Liu, Cornell University)



Definitive Sketch

```
C
C          THIS PROGRAM CALCULATES THE WAVE AMPLITUDE, AND DIRECTION
C          OF WAVE PROPAGATION AT A CROSS SECTION     X = X1.     THE TIP
C          OF THE BREAKWATER IS LOCATED AT     X = X0,    Y = Y0.
C          THE ASYMPTOTICALLY VALID ASYMPTOTIC THEORY USED IN THE ANALYTICAL
C          SOLUTION MUST ASSUMES THAT THE BREAKWATER COINCIDES WITH A
C          HALF RAY RADIATED FROM THE TIP OF THE BREAKWATER.  HOWEVER,
C          THIS PARTICULAR VERSION OF THE PROGRAM HAS BEEN MODIFIED TO
C            REFER TO THE MORE USUAL PLANFORM OF SHORE-CONNECTED
C          BREAKWATER AT AN ANGLE TO THE SHORELINE,  ●●●● A STRAIGHT LINE ●●●●
C          THE ANGLE OF THE RAY AT THE TIP IS DESIGNATED AS ABR.
C
      DIMENSION X(16),FI(2,2),GAUSS(2,8)
      DIMENSION AMP(110),ARE(110),GW(110)
      DIMENSION AM(110),SPE(110),YRE(110)
      DIMENSION TF(110)
      REAL*8 FD,F,XX
      FI(X)=(EXP(X)-EXP(-X))/2.
      SI(X)=(EXP(X)+EXP(-X))/2.
      TA(X)=SI(X)/FI(X)
      F(XX,X)=FI(X)/SI(X)
      FD(XX,X)=SI(X)/SI(X)*(1.-XX*X)
      GAUSS(1,1)=.
      GAUSS(1,2)=.
      GAUSS(1,3)=.
      GAUSS(1,4)=.
      GAUSS(1,5)=.
      GAUSS(1,6)=.
      GAUSS(1,7)=.
      GAUSS(1,8)=.
```

```
C----------THE INPUT DATA FOLLOWS----------
      AI=15.
      ABR=21.
      AG=1.
      A=2.
      A=.2.
      T=.5
      X=.1.
      X=1.
      STP=.2
      X=1.
C
      WRITE (3,*)
   66 FORMAT (1H1,'ACCESS FACILITY FOR SHORE CONNECTED BREAK
     1  WATER')
      WRITE (3,63)
   63 FORMAT (1H0,'NUMERICAL INTEGRATION BY 16 POINT GAUSS
     1 QUADRATURE')
      IF (S .LT. 30.) WRITE (3,71)
   71 FORMAT (1H0,'DISTANCES IN METERS')
      IF (S .GT. 30.) WRITE (3,72)
   72 FORMAT (1H0,'DISTANCES IN FEET')
      PI=3.14159265359
      K0=INT(X0/STP)
      K2=INT(X2/STP)
      KSEA=K2-K0
      S0(K0)=0.
      YBW(K0)=0.
      ABW(K0)=ABR*PI/180.
      DO 333 IAG=1,1
C----------THE NEXT CARD DETERMINES THE INCIDENT WAVE DIRECTION--------
      BI=AI*PI
C----------THE NEXT TWO CARDS DETERMINE THE AMOUNT OF COMPUTED OUTPUT--
C
C
      WRITE (3,51)
   51 FORMAT (1H1,'THIS IS IT')
      WRITE (3,62)
   62 FORMAT (1H0,'      WAVE AT INF.      PERIOD      X OF TIP
     1        X OF EFFECTIVE TIP')
      WRITE (3,*) AI,AG,T
  111 FORMAT (1H0,'
C----------
C
C
C----------RADIAN FREQUENCY----------
C
C----------WAVE NUMBER AT FAR FIELD (DEEP WATER)-----
C----------WAVE NUMBER AT TIP----------
   81 H=DEPTH/X0
  554 P1=P-(G*P*TANH(P*H)-F2)/(G*TANH(P*H)+G*P*H/(COSH(P*H)
     1**2))
      IF (ABS(P-P1) .LT. .0001) GO TO 5551
C
C
 5551 WN1=P1
```

```
      STI=WNI*SIN(AI/180.*PI)
 82   [illegible]
 4    [illegible]NH(P*H)-F2)/(G*TANH(P*H)+G*P*H/(COSH(P*H))
     1 [illegible]
      IF (ABS((P1-P)/P) .LT. 0.005) GO TO 5
      P=P1
      GO TO 4
 5    WNT=P1
      AT=AR[illegible](SCI/WNT)*180./PI
      [illegible]
      [illegible])-AT*PI/180.
      [illegible]
C-----------[illegible]ATE THE LOCATION OF THE BREAKWATER, YBW,----
C-----------[illegible] OF THE PHASE FUNCTION OF THE INCIDENT WAVE,SO,
C-----------AND THE DIRECTIONS OF THE REFLECTED WAVES, AREF,----------
      RAXI[illegible]/SQRT(WNT**2-SCD**2)*(-1.)*STP
      [illegible]*(-1.)*SQRT(WNT**2-SCI**2)
      [illegible]
      [illegible]
      IF [illegible] .NE. [illegible] .OR. MX .NE. 1) GO TO 2001
      WRITE [illegible]
 661  FORMAT [illegible] THE LOCATION OF THE BREAKWATER  ')
      WRITE [illegible],662)
 662  FORMAT [illegible]    X-COORDINATE        Y-COORDINATE')
      [illegible]
      [illegible]
      IF [illegible] NN=KD+L
      IF [illegible] NN=KD-L
      [illegible](L*DC+KD)))
      [illegible] TO 444
 [illegible] TANH(P*H)-F2)/(G*TANH(P*H)+G*P*H/
     1 [illegible]
      IF (ABS((P1-P)/P) .LT. 0.005) GO TO 555
      P=P1
      GO TO [illegible]
 555  IF [illegible] .GT. 1.) GO TO 44[illegible]
      [illegible]**2-SCD**2)*.5*STP*SCD
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]**2-SCI**2)*0.5*STP
      [illegible]+ABS(X[illegible])/(X[illegible]-X)+YBW(NN)*SCI
      [illegible]
      [illegible]
      AR[illegible]BW(NN)-A[illegible]
      WRITE [illegible]
      [illegible]**2-SCD**2)*.5*SCD*STP
      [illegible]**2-SCI**2)*.5*STP
 [illegible]
 44[illegible]
 552  [illegible]'-----------------------------------')
      [illegible]**2-SCD**2)*STP
      [illegible]
 444  [illegible]
      [illegible]
 2001 [illegible]
C----------- [illegible] PHASE FUNCTIONS OF THE REFLECTED
C----------- [illegible] CROSS SECTION XI--------------------------
      [illegible]
 771  [illegible])/(G*TANH(P*H)+G*P*H/
```

```
     1    COSH(P*H)**2)
          IF (ABS((P1-P)/P) .LT. 0.005) GO TO 772
          P=P1
          GO TO 771
  772     SCD=P1*SIN(AREF(NN))
          REFK(NN)=SCD
          IF (NN .EQ. 1) S1=SCD
          IF (NN .EQ. K2) S<2=SCD
          IF (K1 .EQ. NN) STPN=X1-X3
          IF (K1 .NE. NN) STPN=ABS((X1-X3)/(K1-NN))
          IF (NN .LE. K1) DC=1.
          IF (NN .GT. K1) DC=-1.
          RAYII=0.5*SCD/SQRT(P1**2-SCD**2)*(-1.)**C*STPN
          PHAII=0.5*STPN*(-1.)*DC*SQRT(P1**2-SC.**2)
          IF (NN .EQ. K1) K=1
          IF (NN .NE. K1) K=(K1-NN)
          IF (K .LT. 0) K=-K
          IF (NN .EQ. K1) X=X1
          IF (X .EQ. X3) GO TO 888
          DO 889 L=1,K
          IF (NN .NE. K1) X=STPN*L*DC+X3
          IF (L .EQ. 1) GO TO 883
          RAYII=RAYII-DC/SQRT(P1**2-SCD**2)*0.5*STPN*SCD
          PHAII=PHAII-DC*SQRT(P1**2-SCD**2)*0.5*STPN
  883     H=DEPTH(X)
  882     P1=P-(G*P*TANH(P*H)-F2)/(G*TANH(P*H)+G*P*H/
     1    COSH(P*H)**2)
          IF (ABS((P1-P)/P) .LT. 0.005) GO TO 881
          P=P1
          GO TO 882
  881     IF ((SCD/P1)**2 .GT. 1.) GO TO 887
          RAYII=RAYII-DC/SQRT(P1**2-SCD**2)*.5*STPN*SCD
          PHAII=PHAII-DC*SQRT(P1**2-SCD**2)*.5*STPN
  889     CONTINUE
          YRE(NN)=YBW(NN)+RAYII
          SRE(NN)=SO(NN)+ABS(PHAII)*ABS(X3-X)/(X3-X)+SCD*(
     1    YRE(NN)-YBW(NN))
          GO TO 779
  888     YRE(NN)=YBW(NN)
          SRE(NN)=SO(NN)
  779     WRITE (10,200) NN,YRE(NN),SRE(NN)
  777     CONTINUE
          GO TO 83
  887     DO 999 J=1,NN
          M=NN-J+1
          MM1=M+1
          YRE(M)=YRE(MM1)+1.
          SRE(M)=SRE(MM1)+SCD*1.
          REFK(M)=SCD
          WRITE (10,200) M,YRE(M),SRE(M)
  999     CONTINUE
          S1=SCD
C--------------------COMPUTE WAVE NUMBER AT THE CROSS SECTION----------
   83     H=DEPTH(X1)
   16     P1=P-(G*P*TANH(P*H)-F2)/(G*TANH(P*H)+G*P*H/COSH(P*H)
     1     **2.)
          IF (ABS((P1-P)/P) .LT. 0.005) GO TO 17
          P=P1
          GO TO 16
   17     WNCS=P1
          ACS=ARSIN(SCI/WNCS)
C--------------------COMPUTE AMPLITUDE RATIO DUE TO REFRACTION---------
   84     AMF=SQRT(ABS(WNCS*COS(AI*PI/180.)/WNI/COS(ARSIN(SCI/WN
     1     CS))/(1.+2.*WNCS*DEPTH(X1)/SINH(2.*WNCS*DEPTH(X1)))))
          AMF=AMF*SQRT(1.+2.*WNI/SINH(2.*WNI))
          WRITE (10,65)
   65     FORMAT ('0','AMPLITUDE RATIO DUE TO REFRACTION ONLY')
          WRITE (10,100) AMF
          WRITE (10,64)
   64     FORMAT ('0','ANGLE A OF INCIDENT RAY AT THE TIP')
```

```
      WRITE (10,100) AT
C---------------------COMPUTE WAVE NO'S AT GAUSS QUAD. X-POINT----------
      DO 6 K=1,8
      KK=0
      XG=(X1+X0)/2.+(X1-X0)/2.*GAUSS(1,K)
    7 H=DEPTH(XG)
    8 P1=P-(G*P*TANH(P*H)-F2)/(G*TANH(P*H)+G*P*H/(COSH(P*H)
     1  **2.)
      IF (ABS((P1-P)/P) .LT. 0.005) GO TO 9
      P=P1
      GO TO 8
    9 IF (KK-1) 10,11,6
   10 WN(2*K-1)=P1
      KK=KK+1
      XG=X1+X0-XG
      GO TO 7
   11 WN(2*K)=P1
    6 CONTINUE
C---------------------CALCULATE INCIDENT WAVE RAY PHASE INTEGRAL FROM
C---------------------TIP TO X-SECTION---------------------------------
      PHAII=0.
      DO 12 L=1,8
      PHAII=PHAII+(COTAN(ARSIN(SCI/WN(2*L-1)))+COTAN(ARSIN(
     1  SCI/WN(2*L))))*SCI*GAUSS(2,L)*(X1-X0)/2.
   12 CONTINUE
      WRITE (10,63)
   63 FORMAT ('0',' ANGLE A (DEG)    Y LOCATION    AMP. RATIO   DIR
     1  . OF WAVES')
C---------------------PREPARE SCALLING LIMIT---------------------------
      M1=A1/AS
      M2=A2/AS
C---------------------ENTER SCANNING LOOP------------------------------
      DO 13 M=M1,M2
      A=AS*M
      IF (A .EQ. 90.) GO TO 13
      IF (A .EQ. 180.) GO TO 13
      IF( A .EQ. 270.) GO TO 13
      IF (A .EQ. ABR) GO TO 13
C---------------------INITIALIZE---------------------------------------
      PHAID=0.
      RAYID=0.
      SCD=WNT*SIN(A*PI/180.)
      ADCS=ARSIN(SCD/WNCS)
C---------------------COMPUTE DIFFRACTED RAY R AND Y INTEGRALS FROM
C---------------------TIP TO X-SECTION---------------------------------
      DO 15 N=1,8
      PHAID=PHAID+(COTAN(ARSIN(SCD/WN(2*N-1)))+COTAN(ARSIN(
     1  SCD/WN(2*N))))*SCD*GAUSS(2,N)*(X1-X0)/2.
      RAYID=RAYID+(TAN(ARSIN(SCD/WN(2*N-1)))+TAN(ARSIN(SCD/
     1  WN(2*N))))*GAUSS(2,N)*(X1-X0)/2.
   15 CONTINUE
      Y=-1.*RAYID
C---------------------COMPUTE R----------------------------------------
      R=ABS(PHAID)+ABS(Y*SCD)
C---------------------COMPUTE S----------------------------------------
      S=ABS(PHAII)*ABS(X0-X1)/(X0-X1)+Y*SCI
      F01=1.
      F02=-1.
C---------------------COMPUTE SB---------------------------------------
      IF (ABR .GT. AT) GO TO 601
      IF (Y .GT. YRE(K2)) SB=SRE(K2)+SK2*(Y-YRE(K2))
      IF (Y .GT. YRE(K2)) REK=SK2
      IF (Y .LT. YRE(1)) SB=SRE(1)+S1*(Y-YRE(1))
      IF (Y .LT. YRE(1)) REK=S1
      IF (Y .GE. YRE(1) .AND. Y .LE. YRE(K2)) GO TO 602
      GO TO 603
  602 DO 604 I=1,K2
      IF (Y .LE. YRE(I)) GO TO 605
  604 CONTINUE
  605 IF (Y .EQ. YRE(I)) SB=SRE(I)
```

```
      IF (Y .EQ. YRE(I)) REK=REFK(I)
      IF (Y .LT. YRE(I)) SB=(Y-YRE(I))*(SRE(I-1)-SRE(I))
     1  /(YRE(I-1)-YRE(I))+SRE(I)
      IF (Y .LT. YRE(I)) REK=(Y-YRE(I))*(REFK(I-1)-REFK(I))/
     1  (REFK(I-1)-REFK(I))+REFK(I)
      GO TO 605
  601 IF (Y .GT. YRE(1)) SB=SRE(1)+S1*(Y-YRE(1))
      IF (Y .GT. YRE(1)) REK=S1
      IF (Y .LT. YRE(K2)) SB=SRE(K2)+SK2*(Y-YRE(K2))
      IF (Y .LT. YRE(K2)) REK=SK2
      IF (Y .GE. YRE(K2) .AND. Y .LE. YRE(1)) GO TO 606
      GO TO 603
  606 DO 607 I=1,K2
      IF (Y .GE. YRE(I)) GO TO 608
  607 CONTINUE
  608 IF (Y .EQ. YRE(I)) SB=SRE(I)
      IF (Y .EQ. YRE(I)) REK=REFK(I)
      IF (Y .GT. YRE(I)) SB=(Y-YRE(I))*(SRE(I-1)-SRE(I))
     1  /(YRE(I-1)-YRE(I))+SRE(I)
      IF (Y .GT. YRE(I)) REK=(Y-YRE(I))*(REFK(I-1)-REFK(I))
     1  /(YRE(I-1)-YRE(I))+REFK(I)
C--------------------ASSIGN SIGN OF THETA, THETAB FOR DIFFERENT REGIONS
C--------------------IN THE FIELD-------------------------------------
  603 IF (AT .GT. ABR) GO TO 633
      IF (A .GT. AT) FD1=-1.
      IF (A .GT. ABR) FD1=1.
      IF (A .GT. ABR) FD2=1.
      IF (A .GT. AREF(K)*180./PI) FD2=-1.
      GO TO 33
  633 IF (A .GT. AREF(K)*180./PI .AND. A .LT. ABR) FD2=1.
      IF (A .GT. ABR .AND. A .LT. AT) FD1=-1.
C--------------------CALCULATE THETA, THETAB--------------------------
   33 THETA=SQRT(R-S)
      IF ((R-SB) .LT. 0.) GO TO 13
      THETAB=SQRT(R-SB)
C--------------------COMPUTE SINES AND COSINES OF S, SB---------------
      ESS=SIN(S)
      ESC=COS(S)
     ESBS=SIN(SB)
      ESBC=COS(SB)
      XX=THETA**2.
C--------------------COMPUTE FRESNEL INTEGRALS VIA SUBROUTINE CS-------
      DO 35 NN=1,2
      CALL CS(C,SS,XX)
      FI(NN,1)=C
      FI(NN,2)=SS
      XX=THETAB**2.
   35 CONTINUE
C--------------------COMPUTE AMPLITUDE RATIO--------------------------
   34 AMR=AMF*SQRT((((ESC+ESBC-ESS-ESBS)/2.+(ESC*FI(1,1)-ESS
     1  *FI(1,2))*FD1+(ESBC*FI(2,1)-ESBS*FI(2,2))*FD2)**2+((ESC
     2  +ESBC+ESS+ESBS)/2.+(ESS*FI(1,1)+ESC*FI(1,2))*FD1+
     3  (ESBS*FI(2,1)+ESBC*FI(2,2))*FD2)**2)/2.)
      AA1=0.5+FD1*FI(1,1)
      AA2=0.5+FD1*FI(1,2)
      B1=0.5+FD2*FI(2,1)
      B2=0.5+FD2*FI(2,2)
      C1=AA1*AA1+AA2*AA2
      C2=B1*B1+B2*B2
      D1=AA1*B1-AA2*B2
      D2=B1*AA2-AA1*B2
      CSS=ESC*ESBC+ESS*ESBS
      SSS=ESS*ESBC-ESC*ESBS
      DSINY=C1*SCI-C2*REK+(SCI-REK)*(D1*CSS-D2*SSS)
      IF (ABS(REK) .GT. ABS(WNCS)) GO TO 9876
      DSINX=C1*SQRT(WNCS*WNCS-SCI*SCI)+C2*SQRT(WNCS*WNCS-REK*REK)
     1  +(SQRT(WNCS*WNCS-SCI*SCI)-SQRT(WNCS*WNCS-REK*REK))*(D1*CSS-D2
     2  *SSS)
      GO TO 8765
 9876 DSINX=SQRT(WNCS*WNCS-SCI*SCI)*(C1*D1*CSS-D2*SSS)
```

```
8765    [illegible]
        [illegible]
        WRITE [illegible] A[illegible],AMW,AMA
C----------------------------[illegible] TRAIN[illegible] AXIS OF SHORE-CONNECTED
C-------------------------[illegible]A[illegible]A[illegible]-------------------------------------------------
        [illegible]
        IF (FE[illegible] 1.) [illegible] 13
        [illegible]
        [illegible]
        [illegible] 9)[illegible]
        [illegible]
 13     [illegible]
        [illegible]
        [illegible]
        [illegible]

        [illegible]
        [illegible]
        [illegible]
        [illegible]
        [illegible]*Z+5.451182E-7)*Z
        [illegible]+1.1[illegible]544E-2)*Z+1.847965
        [illegible]
        [illegible]*Z+5.[illegible]1141E-6)*
        [illegible]*Z+6.[illegible]E-5)*Z+8.[illegible]
        RETURN
 [illegible] DD=[illegible]
        [illegible]
        Z=[illegible]
        AA=[illegible]E-3)*Z+7.97[illegible]43E-3)*Z-
        [illegible]+5.972151E-3)*Z-1.696428E-5)
        [illegible]*Z-4.444391E-9
        [illegible]E-4*Z+3.401439E-3)*Z-7.27169[illegible]E-5)*Z+7
        [illegible]*Z-9.314910E-3)*Z-1.207998E-6)*Z
        [illegible]
        [illegible]
        [illegible]
        [illegible]*(SS*AA-DD*3)
        RETURN
        END
```

[illegible] Program Output

[illegible]	[illegible]	[illegible]	[illegible] OF WAVES
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	1.191	187.[illegible]39

99.000	27.062	1.177	190.216
100.000	26.225	1.215	189.477
101.000	25.428	1.267	190.000
102.000	24.667	1.291	189.693
103.000	23.940	1.275	188.969
104.000	23.245	1.230	188.713
105.000	22.578	1.183	188.959
106.000	21.938	1.160	189.326
107.000	21.323	1.171	189.507
108.000	20.731	1.210	189.524
109.000	20.161	1.257	189.575
110.000	19.612	1.295	189.741
111.000	19.082	1.314	189.802
112.000	18.569	1.310	189.359
113.000	18.073	1.285	188.533
114.000	17.594	1.245	188.436
115.000	17.129	1.200	189.547
116.000	16.678	1.159	189.947
117.000	16.240	1.131	189.135
118.000	15.815	1.119	190.225
119.000	15.402	1.127	190.862
120.000	15.000	1.150	189.415
121.000	14.608	1.186	190.823
122.000	14.227	1.228	189.951
123.000	13.855	1.271	189.695
124.000	13.492	1.311	190.316
125.000	13.138	1.345	188.971
126.000	12.792	1.372	190.147
127.000	12.454	1.391	188.600
128.000	12.124	1.400	189.817
129.000	11.800	1.401	188.351
130.000	11.483	1.395	189.532
131.000	11.173	1.380	188.207
132.000	10.869	1.360	189.276
133.000	10.571	1.334	188.285
134.000	10.278	1.303	188.920
135.000	9.990	1.269	188.700
136.000	9.708	1.231	188.419
137.000	9.431	1.192	189.291
138.000	9.158	1.151	188.141
139.000	8.889	1.109	189.427
140.000	8.625	1.066	188.742
141.000	8.365	1.024	188.708
142.000	8.109	0.982	189.854
143.000	7.856	0.941	188.394
144.000	7.607	0.901	189.568
145.000	7.362	0.861	189.911
146.000	7.119	0.824	188.303
147.000	6.880	0.787	190.308
148.000	6.644	0.752	189.963
149.000	6.410	0.719	188.117
150.000	6.180	0.687	190.849

151.000	5.951	0.657	190.272
152.000	5.725	0.628	187.550
153.000	5.502	0.601	190.958
154.000	5.281	0.575	191.175
155.000	5.061	0.551	186.741
156.000	4.844	0.528	189.970
157.000	4.629	0.506	192.775
158.000	4.416	0.486	186.959
159.000	4.204	0.466	187.290
160.000	3.994	0.448	193.874
161.000	3.785	0.432	190.259
162.000	3.578	0.416	184.885
163.000	3.372	0.401	192.041
164.000	3.167	0.387	195.523
165.000	2.964	0.374	188.374
166.000	2.761	0.361	188.658
167.000	2.560	0.350	197.111
168.000	2.359	0.339	196.803
169.000	2.160	0.329	192.096
170.000	1.961	0.319	195.426
171.000	1.763	0.310	199.444
172.000	1.565	0.302	198.447
173.000	1.369	0.294	198.702
174.000	1.172	0.286	198.672
175.000	0.976	0.279	194.461
176.000	0.780	0.273	195.521
177.000	0.585	0.266	201.190
178.000	0.390	0.261	195.506
179.000	0.195	0.255	182.298
181.000	-0.195	0.245	201.580
182.000	-0.390	0.241	197.383
183.000	-0.585	0.236	186.856
184.000	-0.780	0.232	195.187
185.000	-0.976	0.228	205.808
186.000	-1.172	0.225	205.254
187.000	-1.369	0.222	205.018
188.000	-1.565	0.218	206.284
189.000	-1.763	0.215	196.671
190.000	-1.961	0.213	188.365
191.000	-2.160	0.210	200.896
192.000	-2.359	0.207	206.106
193.000	-2.560	0.205	191.716
194.000	-2.761	0.203	188.340
195.000	-2.964	0.201	205.698
196.000	-3.167	0.199	210.720
197.000	-3.372	0.197	208.668
198.000	-3.578	0.196	212.182
199.000	-3.785	0.194	204.668
200.000	-3.994	0.193	180.818
201.000	-4.204	0.192	189.190
202.000	-4.416	0.191	211.458
203.000	-4.629	0.190	212.071

204.000	-4.844	0.189	210.642
205.000	-5.061	0.188	213.792
206.000	-5.281	0.188	198.754
207.000	-5.502	0.187	186.079
208.000	-5.725	0.187	208.287
209.000	-5.951	0.187	214.457

Definition of Program Parameters

Name	Description
A	Scanning angle
ABR	Angle of tip of breakwater with normal to shoreline
ABW(110)	Local angle of breakwater
ACS	Angle of incidence at arbitrary cross section
ADCS	Angle of radiated ray at arbitrary cross section
AI	Angle of incidence at infinity (deep water)
AMF	Amplitude ratio due to shoaling effect
AMR	Amplitude ratio due to combined refraction and diffraction
APA	Local angle of wave propagation
AREF(110)	Angle of reflected wave along the breakwater
AS	Scanning angle step
AT	Angle of incidence at the tip of the breakwater
A1	Lower limit of the scanning angle ($\geq$90 deg)
A2	Upper limit of the scanning angle ($\leq$270 deg)
CS	Fresnel integral subroutine
DEPTH(X)	Water depth
ESBC	Cos(SB) (Cosine of phase function of reflected wave ray)
ESBS	Sin(SB) (Sine of phase function of reflected wave ray)
ESC	Cos(S) (Cosine of phase function of incident wave ray)
ESS	Sin(S) (Sine of phase function of incident wave ray)
FI(2,2)	Output for subroutine CS
G	Gravitational acceleration constant, 32.15184 ft/sec^2
GAUSS(2,8)	Location and weight factor for 16-point Gauss Quadrature
H	Water depth
PHAID	Integral part of phase function of radiated wave ray
PHAII	Integral part of phase function of incident wave ray
PI	3.14159265359

Name	Description
R	Phase function of radiated wave ray
RAYID	Integral part of the location of a radiated wave ray
RAYII	Integral part of the location of a reflected wave ray
S	Phase function of incident wave ray
SB	Phase function of reflected wave ray
SCD	Snell's constant for radiated wave ray from breakwater tip
SCI	Snell's constant for incident wave ray
SO(110)	Phase functions of incident waves along the breakwater
SRE(110)	Phase function of reflected wave along X1 cross section
STP	Step size of numerical integration along a ray by the trapezoidal rule
T	Wave period in seconds
THETA	Square root of (R - S)
THETAB	Square root of (R - SB)
WN(16)	Wave number at Gauss Quadrature points
WNCS	Wave number at arbitrary cross section
WNI	Wave number at infinity (deep water)
WNT	Wave number at tip of breakwater
X0	X-coordinate of tip of breakwater
X1	X-coordinate of arbitrary cross section
X2	X-coordinate of end of tracing region (X2 > X0)
XG	X-coordinate of Gauss Quadrature points
YBW(110)	Y-coordinates of the breakwater
YRE(110)	Y-coordinates of reflected wave rays along X1 arbitrary cross section

END

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